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2004 . (

3.)

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1.	5
1.1.	5
1.2.	8
1.3.	11
1.4.	16
1.5.	19
1.6	22
1.7.	27
1.8.	35
1.9.	38
1.10.	43
1.11.	51
2.		
	55
2.1.	55
2.2.	57
2.3.	62
2.4.	64
2.5.	68
2.6.	80
2.7.	89
2.8.		
	100
		112

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108 .

(320).

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Ω
 (\emptyset)
 $\{ \quad_1, \quad_2 ,..., \quad_n \}$
 $\sum_{i=1}^n A_i = U$
 $U,$

[5, . 7-15; 13, . 12-29; 10, . 21-30].

1.
 $\Omega = \{ \omega_i \}, \quad (\omega_i = \ll$
 $i \gg (i = 1, 2,..., 6).$
 $= \ll \quad \gg;$
 $= \ll \quad \gg;$
 $= \ll \quad , \quad 3 \gg.$
 ω_i
 $(\quad).$
 $\omega_2, \quad \omega_4, \quad \omega_6.$
 $A = \{ \omega_2, \omega_4, \omega_6 \}.$
 $B = \{ \omega_1, \omega_3, \omega_5 \}, \ C = \{ \omega_3, \omega_6 \}.$
 Ω

2. -
.

$$\omega = (\quad , \quad)$$

$$\Omega = \{\omega\} = \{(x, y): -a < x < a, \quad -a < y < a\}.$$

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= «
 $r \gg (r <)$.

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« »,
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Ω ,
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1.
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2.
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3. Ω .
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« ».

1.2.

$\omega_i \in A$, ω ,
 $\omega_i \notin A - \omega$.
 \subset ,
 $A \subset B$, а $B \subset A$, (,
) = .
 , = +
 $= A \cup$, ,
 . ,
 $\dots = \{\omega\}$, $= \{\varphi\}$,

$$C = \{\omega\} \cup \{\varphi\}.$$

$= -$,
 ,
 $=$ $= \cap$,
 $= \{\omega\}$, $= \{\varphi\}$,
 ,
 () $\{A_k\}, (k = \overline{1, n})$:

$$C = A_1 + A_2 + \dots + A_k = \sum_{k=1}^n A_k, \quad C = A_1 \cdot A_2 \cdot \dots \cdot A_k = \prod_{k=1}^n A_k.$$

C \overline{A} .
 .

- | | |
|---|--|
| 1. $\quad + \quad =$ | 10. $A + U = U$ |
| 2. $\quad + \quad = \quad +$ | 11. $A \cdot V = V$ |
| 3. $\quad + (\quad + \quad) = (\quad + \quad) +$ | 12. $A \cdot U = A$ |
| 4. $\quad =$ | 13. $\overline{A+B} = \overline{A} + \overline{B}$ |
| 5. $\quad =$ | 14. $\overline{A+B} = \overline{AB}$ |
| 6. $(\quad) = (\quad)$ | 15. $\overline{U} = V$ |
| 7. $(\quad + \quad) = \quad +$ | 16. $A + \overline{A} = U$ |
| 8. $\quad + (\quad) = (\quad + \quad)(\quad + \quad)$ | 17. $A \cdot \overline{A} = V$ |
| 9. $\quad + V = A$ | 18. $\overline{\overline{A}} = A$ |

S
 $\Omega \in S_A$.
 $A \in S_A, B \in S_A, AB \in S_A, A+B \in S_A,$
 $A \setminus B \in S_A$.
 $\sigma, A_n \in S_A,$
 $n=1,2,\dots, \bigcup_{n=1}^{\infty} A_n \in S_A, \bigcap_{n=1}^{\infty} A_n \in S_A$.

[5, . 11-14; 13, . 18-27; 10, . 21-26].

1. , $-$,
 , , , (\quad) :
) ;
) ;
) ;
) , .

1. $\cdot = \ll$
 $\gg; = \ll$ \gg . $: \bar{A}, \bar{B}$,
 $+ , \cdot \bar{B}, \bar{A} + \bar{B}, \cdot , \bar{A} \cdot , (\bar{A} \cdot \bar{B})$.
2. $\frac{25}{20}$ (), $9 -$
 $() 6 -$ (\cdot).
 $\cdot \bar{B}, \bar{A} \cdot , \overline{A+B}$.
- 3.

$$(\quad + \quad) \cdot (\quad + \bar{B}) \cdot (\bar{A} + \quad).$$

4. $\overline{A \cdot B}?$

1.3.

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 \cdot ,
 $($
 $)$ $()$
 $() - \left(\frac{m}{n}\right)$.
 $\left(\frac{m}{n}\right)$,
 $,$,
 \dots
 \cdot ,
 24000 12012
 $\frac{m}{n} = \frac{12012}{24000} = 0,5005 \dots$
 $\left(\frac{m}{n}\right) 0,5$.
 $,$,
 \cdot .

$$\bullet \quad \left(\begin{array}{c} \text{ } \\ \text{ } \end{array} \right) \left(\begin{array}{c} \text{ } \\ \text{ } \end{array} \right)$$

,

$$P(A)=\frac{m}{n}. \quad (1.1)$$

$$, \quad \Omega$$
$$(\quad)^*$$
$$(\Omega^*)_{n-1} = \{ \omega \in \Omega^* : \omega \text{ is } n-1 \text{ dimensional} \}, \quad (2.1.1)$$

$$P(A) = \frac{\mu(A^*)}{\mu(\Omega^*)}, \quad (1.2)$$

$$\mu(A^*), \mu(\Omega^*) - \int_{\Omega^*} \mu^*(x) dx = \int_{\Omega^*} (\mu(x) - \mu^*(x)) dx,$$

. . . (1903-1987.)

$$(\quad) \quad S_\sigma,$$

σ_- (, .1.2) .

$$\sigma^- \in S_\sigma, \quad (1)$$

,

$$(\quad):$$

1. $0 \leq P(A) \leq 1$.

2. $P(U) = 1$.

3. $A_i \in S_\sigma \quad (n = 1, 2, \dots)$ $(A_i \cdot A_j = V,$

$$i \neq j; i, j = 1, 2, \dots),$$

$$P\left(\sum_{i=1}^{\infty} A_i\right)=\sum_{i=1}^{\infty} P(A_i).$$

$(\Omega, \mathcal{S}_\sigma, \mathbb{P}(\cdot)), \mathcal{S}_\sigma$
 σ - (\cdot) ,
 \cdot .

[5, . 15-30; 13, . 29-37; 10, . 26-36].

1. , 1000
 \cdot .
 \cdot ,
 \cdot .
12 , 8
 \cdot ,
 (\cdot)
 $= 8 \cdot 12 = 96.$ $= 1000.$

$$P(A)=\frac{m}{n}=\frac{96}{1000}=0,096.$$

2. ,
 \cdot ,
 \cdot ?

$n = C_{27}^3$ (
27 3). = « »

$$m = C_{15}^2 \cdot C_{12}^1, \quad C_{15}^2 -$$

15, 2

$$C_{12}^1 = 12 -$$

$$P(A) = \frac{m}{n} = \frac{C_{15}^2 \cdot C_{12}^1}{C_{27}^3} = 0,4308$$

3.

$[0, 2]$

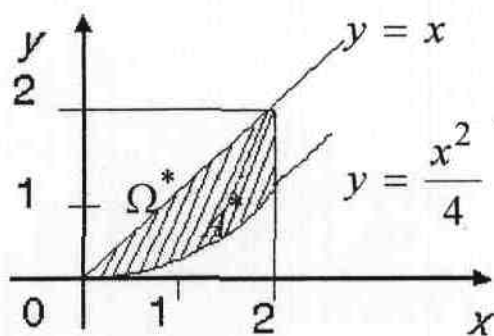
$$x^2 \leq 4y \leq 4x.$$

(,)

*,

(Ω^*),

$$x^2 \leq 4y \leq 4x. \quad (1.1).$$



. 1.1

= «

$[0, 2]$

$$x^2 \leq 4y \leq 4x \gg$$

,

(,)

(1.2)

:

Ω^*

$$P(A) = \frac{\int_0^2 \left(x - \frac{x^2}{4} \right) dx}{4} = \frac{1}{3}.$$

4.

40-

70-

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45-

50-

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1.4.

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- (/).

, $P(A/B) = P(A)$.

$$() = () (/) = () (/), \quad (1.4)$$

:

$$P\left(\prod_{i=1}^n A_i\right) = P(A_1)P(A_2 / A_1)P(A_3 / A_1A_2)...P\left(A_n / \sum_{i=1}^{n-1} A_i\right) \quad (1.5)$$

1, 2,..., n

,

(= 2,3,...,)

 $k_j (j = 1,2,...,n) \ 1 \leq k_1 < k_2 < ... < k_m \leq n$

$$P\left(\prod_{j=1}^m A_{k_j}\right) = \prod_{j=1}^m P(A_{k_j}).$$

$$(+) = () + () - (). \quad (1.6)$$

,

$$\begin{aligned} (+ +) &= (+ (+)) = () + (+) - P(A(B +)) = \\ &= () + () + (C) - P() - (+) = \\ &= () + () + P(C) - P(BC) - P(AB) - () + (), \end{aligned}$$

.

$P(A) = 1 - P(\bar{A}) = \frac{C_n^k}{C_n^k} = 1$

$$P(\bar{A}) = \frac{C_{n-m}^k}{C_n^k}; \quad P(A) = 1 - \frac{C_{n-m}^k}{C_n^k}.$$

1. $P(A) = \frac{1}{2}, P(B) = \frac{2}{3}$

2. k -
 $(k = 1, 2, \dots, n)$.

3. 0,2.

4. 0,012; 0,010; 0,006 0,002.

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5. « X » .

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« »?

1.5.

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()

. ($i = \overline{1, n}$) –

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$P(H)$, ($/H$).

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$$P(A) = \sum_{i=1}^n P(H_i)P(A/H_i). \quad (1.7)$$

.

,

()

$P(H_i/A)$

.

$$P(H_i/A) = \frac{P(H_i) \cdot P(A/H_i)}{P(A)} \quad (1.8)$$

() (1.7).

[5, . 34-40; 13, . 46-52; 10, . 48-52].

50 . , , , , 0,7; 0,8; 0,9. , ?

= « (= 1,2,3) ,

$$P(H_1) = \frac{20}{100} = 0,2, \quad P(H_2) = \frac{30}{100} = 0,3, \quad P(H_3) = \frac{50}{100} = 0,5.$$

$$P(A/H_1) = 0,7, \quad P(A/H_2) = 0,8, \quad P(A/H_3) = 0,9. \quad (1.7)$$

$$P(A) = \sum_{i=1}^3 P(H_i)P(A/H_i) = 0,2 \cdot 0,7 + 0,3 \cdot 0,8 + 0,5 \cdot 0,9 = 0,83$$

3. «0» «1». 2/5 «0» 1/3 «1». «0» «1» 5:3.

:) «0»;) «1».

= « «0»», = « «1»». : 1 - «0», 2 - «1». (1)/ (2) = 5 / 3. (1) + (2) = 1 - (1) = 5 / 8, (2) = 3 / 8. (/ 1) = 2 / 5 («1» «0»), (/ 2) = 3 / 5.

$$P(A/H_2) = 1/3 \Rightarrow P(B/H_2) = 2/3.$$

1. 20 10 . 4 3

2. , ? . ?

3. , .

4. , 1000 100 , 1000 0 5.

1.6. .

(1654-1705) : () = (0 < < 1). .
 $P_n(m) (0 \leq \leq)$

$$P_n(m)=C_n^m p^m q^{n-m}, \tag{1.9}$$

$q=P(\overline{A})=1-p.$
 $P_n(m) (= 0,1,2,...,n)$, . .

(1.9)

$$(p+q)^n = \sum_{m=0}^n C_n^m p^m q^{n-m}$$

,

1.

(1.9)

$$\left(\begin{array}{c} P_n(m) \end{array} \right).$$

,

.

-

$$P_n(m) \approx \frac{1}{\sqrt{npq}} \varphi(x), \quad (1.10)$$

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}; \quad x = \frac{m - np}{\sqrt{npq}}.$$

-

,

$$\in (0;1)$$

$$P_n(m_1 \leq m \leq m_2) \approx \frac{1}{\sqrt{2\pi}} \int_{x_1}^{x_2} e^{-\frac{x^2}{2}} dx = \Phi_0(x_2) - \Phi_0(x_1) \quad (1.11)$$

$$x_i = \frac{m_i - np}{\sqrt{npq}}, \quad i = 1, 2.$$

(1.11) (

)

(

).

$$\Phi_0(x) = \frac{2}{\sqrt{2\pi}} \int_0^x e^{-\frac{t^2}{2}} dt, \quad x \in R,$$

:

✓ $\Phi_0(x) -$

✓ $\Phi_0(x) - R;$

✓ $\lim_{x \rightarrow +\infty} \Phi_0(x) = 1;$

✓ $\lim_{x \rightarrow -\infty} \Phi_0(x) = -1;$

✓ $x \geq 4 \quad \Phi_0(x) = 0,9998\dots$

$\Phi_0(x)$ (. . 1)

$\rightarrow \infty,$

$\rightarrow 0$, $\rightarrow \lambda, \quad 0 < \lambda < \infty,$

$$P_n(m) \approx \frac{\lambda^m}{m!} e^{-\lambda}.$$

(1.12)

[5, . 41-51; 13, . 57-75; 10, . 52-67].

1.

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0,6.

8

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.

(1.9)

,

$P_8(2) = C_8^2(0,6)^2(0,4)^2 \approx 0,041.$

$-q \leq m_0 \leq +q.$

$$8 \cdot 0,6 - 0,4 \leq m_0 \leq 8 \cdot 0,6 + 0,4 \quad \Rightarrow \quad m_0 = 5,$$

$$P_8(5) = C_8^5 (0,6)^5 \cdot (0,4)^3 \approx 0,28.$$

2.

$$\begin{array}{ccc} & & , \\ & 0,005. & , \\ 10\,000 & & 70? \end{array}$$

$$P_{10000}(0 \leq m \leq 70) = \sum_{m=0}^{70} C_{10000}^m (0,005)^m (0,995)^{10000-m}$$

$$\begin{array}{ccc} , & & . \\ (1.11). & & _1 = 0, \quad _2 = 70, n = 10000, \\ = 0,005, q = 0,995. & & \end{array}$$

$$x_1 = \frac{m_1 - np}{\sqrt{npq}} \approx 7,09, \quad x_2 = \frac{m_2 - np}{\sqrt{npq}} \approx 2,84.$$

$$\Phi_0(x)$$

:

$$P_{10000}(0 \leq m \leq 70) \approx \frac{1}{2} (\Phi_0(2,84) + \Phi_0(7,09)) \approx 0,4977 + 0,5 \approx 0,9977.$$

3.

$$\begin{array}{ccc} 800 & , & 0,005. \\ & & . \end{array}$$

$$, \quad .$$

$$-q \leq m_0 \leq +q \quad \Rightarrow \quad m_0 = 4.$$

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2.

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0,7.

3.

1%
200

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4.

100 000 .
0,0001.

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5.

10 000 .

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0,7. ,

5 900 6 100.

1.7.

Ω ,

ω (ω).

ξ

$\xi(\omega)$,

Ω

$$\begin{pmatrix} a_1 & a_2 & \dots \\ P_1 & P_2 & \dots \end{pmatrix},$$

ξ ,

$= (\xi =)$, ξ ,

,

$$\sum_{a_i} P(\xi = a_i) = 1.$$

[5, . 58-65; 13, . 64; 10, . 73].

1.

$$P(\quad) = \quad.$$

I_A ,

.

$$I_A = 1 - \quad, \quad \ll \quad \gg.$$

$$I_A = 0 - \quad, \quad \ll \quad \gg.$$

,

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I_A	0	1
$p(I_A)$	$1-p$	p

2.

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2

8

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ξ

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ξ

1, 2, 3.

$(\xi = 1) -$

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$$p(\xi = 1) = \frac{8}{10} = \frac{4}{5},$$

$(\xi = 2) -$

,

, . .

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,

$$p(\xi = 2) = \frac{2}{10} \cdot \frac{8}{9} = \frac{8}{45},$$

$(\xi = 3) -$

,

,

$$p(\xi = 3) = \frac{2}{10} \cdot \frac{1}{9} = \frac{1}{45}.$$

ξ	1	2	3
$p(\xi)$	$\frac{4}{5}$	$\frac{8}{45}$	$\frac{1}{45}$

3. , $\frac{4}{5} \cdot \xi -$

.

$k-$, $k-1$
 , k . $P(\xi = k) = \left(\frac{4}{5}\right)^{k-1} \cdot \frac{1}{5},$

$k = 1, 2, \dots$. :

ξ	1	2	...	k	...
$p(\xi)$	$\frac{1}{5}$	$\frac{4}{5} \cdot \frac{1}{5}$...	$\left(\frac{4}{5}\right)^{k-1} \cdot \frac{1}{5}$...

,

,

.

4. .

,

$\frac{1}{2}.$

$\xi -$

0;1; 2; 3,4.

$p(\xi = m)$

$$P(\xi = m) = \frac{n!}{m!(n-m)!} p^m q^{n-m},$$

$n = 4, P = 0,5, q = 0,5, m = 0, 1, 2, 3, 4.$

ξ

ξ	0	1	2	3	4
$p(\xi)$	0,0625	0,25	0,375	0,25	0,0625

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$M_1(x_1, P_1), M_2(x_2, P_2), \dots, M_n(x_n, P_n), \dots,$ $x_i -$

, $P_i -$

.

$\xi.$

1.

.

$A = \ll$

$\gg, B = \ll$

$3\gg.$

$$I_A, I_B.$$

2. 28

ξ

3.

0,4.

4.

25

3

6

ξ

ξ

(x_1, \quad_2)

$$P(x_1 < \xi < x_2) = \int_{x_1}^{x_2} f(x) dx$$

$$f(x)$$

$$\int_{-\infty}^{+\infty} f(x) dx = 1.$$

$$f(x)$$

ξ .

[5, . 69-72; 13, . 83-86; 10, . 111-119].

1. ξ

$$f(x) = \begin{cases} 0 & x \leq 0, \\ \frac{1}{2} \sin x & 0 < x \leq \pi, \\ 0 & x > \pi. \end{cases}$$

$$\left(0; \frac{\pi}{4}\right).$$

$$P\left(0 < \xi < \frac{\pi}{4}\right) = \frac{1}{2} \int_0^{\frac{\pi}{4}} \sin x dx$$

$$P\left(0 < \xi < \frac{\pi}{4}\right) = -\frac{\cos x}{2} \Big|_0^{\frac{\pi}{4}} = \frac{1}{2} \left(1 - \frac{\sqrt{2}}{2}\right).$$

2.

$$f(x) = \frac{A}{e^x + e^{-x}},$$

$$f(x)$$

$$A \int_{-\infty}^{+\infty} \frac{1}{e^x + e^{-x}} dx = 1.$$

$$A \int_{-\infty}^{+\infty} \frac{1}{e^x + e^{-x}} dx = A \operatorname{arctg} e^x \Big|_{-\infty}^{+\infty} = A \left(\frac{\pi}{2} - 0 \right),$$

$$\frac{A\pi}{2} = 1, \quad A = \frac{2}{\pi}.$$

)

$$P(-\infty < \xi < 1) = \frac{2}{\pi} \int_{-\infty}^1 \frac{e^x dx}{e^{2x} + 1} = \frac{2}{\pi} \operatorname{arctg} e.$$

$$\frac{2}{\pi} \operatorname{arctg} e - \quad , \quad \xi$$

$$\left(\frac{2}{\pi} \operatorname{arctg} e \right)^2 \approx 0,6015.$$

$$3. \quad \xi$$

$$, \sigma \ (\sigma > 0),$$

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-a)^2}{2\sigma^2}}.$$

ξ

$$P(x_1 < \xi < x_2) = \frac{1}{2} \left(\Phi \left(\frac{x_2 - a}{\sigma} \right) - \Phi \left(\frac{x_1 - a}{\sigma} \right) \right),$$

$$\Phi(x) = \frac{2}{\sqrt{2\pi}} \int_0^x e^{-\frac{t^2}{2}} dt$$

$$\Phi(x) \quad . \quad 1.6.$$

$$\xi, \sigma (\sigma > 0) \quad a - 3\sigma < \xi < a + 3\sigma .$$

$$P(a - 3\sigma < \xi < a + 3\sigma) = \frac{1}{2} \left(\Phi \left(\frac{a + 3\sigma - a}{\sigma} \right) - \Phi \left(\frac{a - 3\sigma - a}{\sigma} \right) \right) = \Phi(3).$$

$$\Phi(3) = 0,9973.$$

$$, \quad \xi \\ (a - 3\sigma; a + 3\sigma).$$

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1. ξ

$$f(x) = \begin{cases} cx^{-\frac{3}{2}} & x \geq 1, \\ 0 & x < 1. \end{cases}$$

.

2. $(\alpha; \beta)$ ξ $f(x)$ ξ $(\alpha_1; \alpha_2),$ $x_1 \in (\alpha; \beta),$

$$x_2 \in (\alpha; \beta).$$

3.

$$= 0, \quad \sigma = 1.$$

$$P(-0,5 \leq \xi \leq -0,1) \qquad P(1 \leq \xi \leq 2)?$$

1.8.

$$\begin{array}{c} \cdot \\ F(x) \\ \xi, \\ , \qquad \xi \qquad \qquad \qquad , \quad \cdot \cdot \end{array}$$

$$F(x) = P(\xi < x).$$

ξ

$$F(x) = \int\limits_{-\infty}^x f(t) dt,$$

$$f(x) - \qquad \qquad \qquad \xi.$$

$$F(x) = f(x),$$

$$f(x).$$

ξ

,

.

:

$$1. \quad 0 \leq F(x) \leq 1.$$

$$2. \quad P(a \leq \xi < b) = F(b) - F(a).$$

$$3. \quad F(x_1) \leq F(x_2), \qquad x_1 < x_2.$$

$$4. \quad F(-\infty) = 0, \quad F(+\infty) = 1.$$

[5, . 65-72; 13, . 77-86; 10, . 111-119].

1.

ξ

ξ	1	2	3
(ξ)	0,3	0,5	0,2

$F(x)$

$\xi.$

$$F(x) = \begin{cases} 0, & \xi \leq 1, \\ 0,3, & 1 < \xi \leq 2, \\ 0,5, & 2 < \xi \leq 3, \\ 1, & \xi > 3. \end{cases}$$

$$\eta(x) = \begin{cases} 0, & \leq 0, \\ 1, & > 0 \end{cases}$$

$$F(x) = 0,3\eta(x-1) + 0,5\eta(x-2) + 0,2\eta(x-3)$$

$$F(x) = \sum_i P_i \eta(x - x_i)$$

2.
(a = 0, σ = 1)

$$F(x)=\frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^xe^{-\frac{t^2}{2}}dt$$

$$F(x)=\frac{1}{2}(\Phi(x)-\Phi(-x))$$

$$\Phi(x)=\frac{2}{\sqrt{2\pi}}\int\limits_0^xe^{-\frac{t^2}{2}}dt;\quad \Phi(-\infty)=-1.$$

$$F(x)=\frac{1}{2}(\Phi(x)+1).$$

1. ξ $f(x)=Ax^2e^{-\lambda x}$,
(x > 0, 0 ≤ x < +∞). ;)

$$\xi \quad \left(0; \frac{1}{\lambda}\right).$$

2.

 ξ

$$F(x) = A + B \operatorname{arctg} x, \quad (-\infty < x < +\infty).$$

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$$f(x);$$

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 ξ

$$[-1; 1].$$

3.

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 ξ

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1

$$\frac{1}{2}.$$

)

$$F(x)$$

 $\xi,$

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4.

 ξ

:

$$F(x) = \begin{cases} 0, & x \leq 0, \\ x, & 0 < x \leq 1, \\ 1, & x > 1. \end{cases}$$

)

 $\xi.$

)

$$F(x)$$

$$f(x).$$

1.9.

$$f(x)$$

 ξ $\eta,$

$$y = f(\xi),$$

 ξ

.

$y = f(\xi)$

ξ

.

ξ	1	2	...
(ξ)	1	2	...

$y = \varphi(\xi)$

,

$\eta = \varphi(\xi)$

η	$\varphi(_1)$	$\varphi(_2)$...
(η)	1	2	...

$y = \varphi(\xi)$

,

,

$a = \varphi(x_i),$

.

,

$P(\varphi(x_i) = a)$

,

ξ

i .

[5, . 73-74; 13, . 48-102; 9, . 63-64].

1.

ξ

ξ	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$
(ξ)	0,2	0,7	0,1

$\eta = \sin \xi.$

η	$\sin \frac{\pi}{4}$	$\sin \frac{\pi}{2}$	$\sin \frac{3\pi}{4} = \sin \frac{\pi}{4}$
(η)	0,2	0,7	0,1

η	$\frac{\sqrt{2}}{2}$	1
(η)	0,3	0,7

ξ

,

ξ .

2. $y = ax + b$.

$$\eta = a\xi + b, \quad > 0,$$

$$F_{\eta}(x) = P(\eta < x) = P(a\xi + b < x) \underset{a>0}{=} P\left(\xi < \frac{x-b}{a}\right) = F_{\xi}\left(\frac{x-b}{a}\right).$$

$$f_{\eta}(x)$$

$$f_{\eta}(x) = F'_{\eta}(x) = \left(F_{\xi}\left(\frac{x-b}{a}\right)\right)'_x = F'_{\xi}\left(\frac{x-b}{a}\right) \frac{1}{a} = f_{\xi}\left(\frac{x-b}{a}\right) \frac{1}{a}.$$

$$< 0,$$

$$F_{\eta}(x) = P(\eta < x) = P(a\xi + b < x) \underset{a<0}{=} P\left(\xi > \frac{x-b}{a}\right) = 1 - F_{\xi}\left(\frac{x-b}{a}\right).$$

$$f_{\eta}(x) = F'_{\eta}(x) = \left(F_{\xi}\left(\frac{x-b}{a}\right)\right)'_x = \frac{-1}{a} f_{\xi}\left(\frac{x-b}{a}\right).$$

,

$$f_{\eta}(x) = \frac{1}{|a|} f_{\xi}\left(\frac{x-b}{a}\right).$$

3.

$$f(x) = x^3.$$

$$\eta = \xi^3.$$

$$F_{\eta}(x) = P(\eta < x) = P(\xi^3 < x) \underset{a < 0}{=} P\left(\xi < x^{\frac{1}{3}}\right) = F_{\xi}\left(x^{\frac{1}{3}}\right).$$

$$f_{\eta}(x) = \left(F_{\xi}(x^{1/3})\right)'_x = F'_{\xi}(x^{1/3}) \cdot \frac{1}{3} x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}} f_{\xi}(x^{1/3})$$

4.

$$f(x) = e^{-x}.$$

 > 0

$$\begin{aligned} F_{\eta}(x) &= P(\eta < x) = P(e^{-\xi} < x) = \\ &= P(\xi > -\ln x) = 1 - P(\xi < -\ln x) = 1 - F_{\xi}(-\ln x), \end{aligned}$$

$$f_{\eta}(x) = F'_{\eta}(x) = -\left(F'_{\xi}(-\ln x)\right)'_x = \frac{1}{x} f_{\xi}(-\ln x).$$

 < 0

$$F_{\eta}(x) = P(\eta < x) = 0, \quad \dots \quad \eta = e^{-\xi} > 0.$$

$$f_{\eta}(x) = F'_{\eta}(x) = 0.$$

$$5. \quad y = x^2. \quad F_{\eta}(x) \quad f_{\eta}(x)$$

$$\eta = \xi^2.$$

$$> 0$$

$$\begin{aligned} F_{\eta}(x) &= P(\eta < x) = P(\xi^2 < x) = P(-\sqrt{x} < \xi < \sqrt{x}) = \\ &= P(\xi < \sqrt{x}) - P(\xi < -\sqrt{x}) = F_{\xi}(\sqrt{x}) - F_{\xi}(-\sqrt{x}), \end{aligned}$$

$$\begin{aligned} f_{\eta}(x) &= F'_{\eta}(x) = F'_{\xi}(\sqrt{x}) \frac{1}{2\sqrt{x}} - F'_{\xi}(-\sqrt{x}) \left(-\frac{1}{2\sqrt{x}} \right) = \\ &= \frac{1}{2\sqrt{x}} (F'_{\xi}(\sqrt{x}) + F'_{\xi}(-\sqrt{x})) = \frac{1}{2\sqrt{x}} f_{\xi}((\sqrt{x}) + (-\sqrt{x})). \end{aligned}$$

$$\leq 0 \quad F_{\eta}(x) = 0 \quad f_{\eta}(x) = 0.$$

.

ξ

$$F_{\xi}(x)$$

$$f_{\xi}(x)$$

$$y = \varphi(x)$$

$$, \quad x = \varphi^{-1}(y)$$

—

.

$$P(\eta < x) = P(\varphi(\xi) < x) = P(\xi < \varphi^{-1}(y)) = F_{\xi}(\varphi^{-1}(y));$$

$$f_{\eta}(x) = \left(F_{\xi}(\varphi^{-1}(y)) \right)'_x.$$

$$y = \varphi(x) - ,$$

$$1 - F'_{\xi}(\varphi^{-1}(y)), \quad f_{\eta}(x) = - \left(F_{\xi}(\varphi^{-1}(y)) \right)'_x.$$

$$[5, \quad .73-74; 9, \quad .63-64; 13, \quad .98-102].$$

1. ξ

ξ	-2	-1	0	1	2
$P_{\xi}(x)$	$0,1$	$0,2$	$0,3$	$0,3$	$0,1$

$\eta = \xi^2 + 1 \quad \xi = |\xi|.$

2.

$$P_{\xi}(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$$

$$\eta = \frac{1}{\xi}.$$

3. ξ $f_{\xi}(x).$
 $\eta = 3\xi.$

4. ξ $\left[-\frac{\pi}{2}; \frac{\pi}{2}\right].$
 $\eta = \sin \xi.$

1.10.

.

Ω

$\xi(\omega) \ (\omega \in)$

$\xi(\omega)$		1	2	...
$P(\xi(\omega))$		1	2	...

$$\sum_{\omega \in \Omega} \xi(\omega)P(\xi(\omega)) \qquad ,$$

$$\xi = \sum_{\omega \in \Omega} \xi(\omega)P(\xi(\omega))$$

ξ .

$$\sum_{\omega \in \Omega} \xi(\omega)P(\xi(\omega))$$

$$\sum_i a_iP_i \qquad \sum_{\omega \in \Omega} \xi(\omega)P(\xi(\omega)) = \sum_i a_iP_i \, .$$

$$, \qquad \sum_i a_iP_i$$

$$\sum_{\omega \in \Omega} \xi(\omega)P(\xi(\omega))$$

$$\sum_i a_iP_i \, = \, \sum_{\omega \in \Omega} \xi(\omega)P(\xi(\omega)) \, .$$

:

ξ

$$(\, \, \, 1, \, \, \, 2, \dots) \qquad \qquad \qquad (\, \, \, 1,$$

$$2, \dots) \qquad \qquad \qquad \sum_i a_iP_i \, .$$

$$[5, \quad . \, 85-89; 9, \quad . \, 108-110; 13, \quad . \, 105-113].$$

ξ

$$f(x).$$

$$\xi = \int\limits_{-\infty}^{+\infty} xf(x)dx \qquad \qquad \qquad (1.16)$$

[5, . 96-98; 9, . 110-114; 13, . 105-115].

$\eta = \varphi(x)$

$$\varphi(\xi) = \sum_i \varphi(x_i) P_i \tag{1.17}$$

$$\xi = \dots; \quad i = \dots \sum_i \varphi(x_i) P_i$$

$$\xi = \dots,$$

$$\varphi(\xi) = \int_{-\infty}^{+\infty} \varphi(x) f(x) dx \tag{1.18}$$

$$f(x) = \dots \xi$$

- 1. $\mathbf{M} = \dots - nst.$
 - 2. $(\xi) = \xi.$
 - 3. $(\xi \pm \eta) = \xi \pm \eta, \quad \xi \eta - \dots$
 - 4. $(\xi \cdot \eta) = \xi \cdot \eta, \quad \xi \eta \dots$
- $\xi \eta \dots,$

$$P(\xi < x, \eta < y) = P(\xi < x) \cdot P(\eta < y).$$

$$[5, \quad .\,87\text{-}88; 9, \quad .\,118\text{-}120; 13, \quad .\,113\text{-}117].$$

$$\xi=\xi\left(\omega \right),\left(\omega \in \Omega \right)$$

$$\mathbf{D}\xi = \mathbf{M}(\xi - \mathbf{M}\xi)^2 \tag{1.19}$$

$$\cdot\cdot$$

$$\cdot$$

$$\sqrt{\mathbf{D}\xi} \\ \xi.$$

$$\mathbf{D}\xi=\sum_i\left(x_i-\mathbf{M}\xi\right)^2P_i,$$

$$-\xi.$$

$$\xi$$

$$\mathbf{D}\xi=\int\limits_{-\infty}^{+\infty}\left(x-\mathbf{M}\xi\right)^2f(x)dx$$

$$f(x)-\cdot$$

$$\mathbf{D}\xi=\mathbf{M}\xi^2-\left(\mathbf{M}\xi\right)^2.$$

$$1.\,\mathbf{D}\,=\,,\quad-\,\,nst.$$

$$2.\,\mathbf{D}\,(\,\,\xi)=\,\,^2\,\mathbf{D}\,\xi,\quad-\,\,nst.$$

$$3.\,\mathbf{D}\,(\xi\pm\eta)=\mathbf{D}\,\xi\pm\mathbf{D}\,\eta,\quad\,\,\xi\,\,\eta-\cdot$$

[5, . 100-106; 9, . 120-125; 13, . 117-121].

1. ,
A.

I_A		
I_A	0	1
$P(I_A)$	$1-P$	P

« » A .

$$I_A = 0(1 -) + 1 \cdot = ,$$

$$\begin{aligned} \mathbf{D} I_A &= (I_A - (I_A))^2 = (0 - P)^2 (1 - P) + (1 - P)^2 P = \\ &= (1 - P)(P^2 + P - P^2) = P(1 - P), \end{aligned}$$

$$\mathbf{D} I_A = p \cdot q$$

$$q = 1 - P,$$

2. $\mathbf{D}\xi$ (ξ), ξ
, . .

$$P(\xi = m) = C_n^m p^m q^{n-m},$$

$n \geq m -$; $-$ « »
A ; $q = 1 - p -$ «
» A .

$A)$ n ξ (« » .

$$\xi = I(A_1) + I(A_2) + \dots + I(A_n).$$

$$I(A_k) = 1 - \frac{k}{n} \quad 0 - \xi \quad n$$

(3)

1

$$\mathbf{M}\xi = \sum_{k=1}^n \mathbf{M}(I_{A_k}) = nP \quad (\dots \mathbf{M}(I_{A_k}) = P).$$

$$I(A_1), I(A_2), \dots, \quad (3)$$

$$\mathbf{D}\xi = \sum_{k=1}^n \mathbf{D}(I_{A_k}), \quad \mathbf{D}(I_{A_k}) = pq, \quad k = 0, 1, \dots, n.$$

$$\mathbf{D}\xi = npq.$$

3.

ξ

$$P(\xi = k) = \frac{\lambda^k e^{-\lambda}}{k!}, \quad k = 0, 1, \dots, n.$$

$$\begin{aligned} \mathbf{M}(\xi) &= \sum_{k=0}^{\infty} k \frac{\lambda^k e^{-\lambda}}{k!} = e^{-\lambda} \cdot \lambda \sum_{k=1}^{\infty} \frac{\lambda^{k-1}}{(k-1)!} = \\ &= e^{-\lambda} \cdot \lambda \cdot e^{\lambda} = \lambda, \end{aligned}$$

..

$$1 + \frac{\lambda}{1!} + \dots + \frac{\lambda^k}{k!} + \dots = e^{\lambda}.$$

$$\mathbf{D}\xi = (\xi - \xi)^2, \quad \xi^2 = \mathbf{M}\xi (\xi - 1) + \xi.$$

$$\mathbf{M}\xi(\xi-1)=\sum_{k=0}^{\infty}k(k-1)\frac{\lambda^k}{k!}e^{-\lambda}=e^{-\lambda}\cdot\lambda^2\sum_{k=2}^{\infty}\frac{\lambda^{k-2}}{(k-2)!}=\lambda^2.$$

$$\mathbf{M}\xi^2=\lambda^2+\lambda\quad \mathbf{D}\xi=\lambda^2+\lambda-\lambda^2=\lambda.$$

$$4. \qquad \qquad \qquad \xi \qquad \qquad \qquad [\quad; \quad b].$$

$$(\xi) \quad \mathbf{D}\xi.$$

$$f(x)=\frac{1}{b-a}, \quad x\in[a;b].$$

$$f(x)=0, \quad x\notin[a;b].$$

$$\mathbf{M}\xi=\int\limits_a^b\frac{xdx}{b-a}=\frac{x^2}{2(b-a)}\Big|_a^b=\frac{b+a}{2}.$$

$$\mathbf{D}\xi \qquad \qquad \qquad \mathbf{D}\xi = \qquad \xi^2 - (\qquad \xi)^2.$$

$$\mathbf{M}\xi^2=\int\limits_a^b\frac{x^2dx}{b-a}=\frac{x^3}{3(b-a)}\Big|_a^b=\frac{b^2+ab+a^2}{3}.$$

$$\mathbf{D}\xi=\frac{b^2+ab+a^2}{3}-\frac{(a+b)^2}{4}=\frac{(b-a)^2}{12}.$$

5.

$$\xi \qquad \qquad \qquad , \quad \dots$$

$$f(x)\!=\!\lambda e^{-\lambda x},\;\;x>0.$$

$$\mathbf{M}\xi=\int\limits_0^{+\infty}x\lambda e^{-\lambda x}dx=\frac{1}{\lambda}\;(\hspace{10cm}).$$

$$\mathbf{D}\xi=\int\limits_0^{+\infty}x^2\lambda e^{-\lambda x}dx-\frac{1}{\lambda^2}=\frac{2}{\lambda^2}-\frac{1}{\lambda^2}=\frac{1}{\lambda^2}.$$

$$\sigma_{\xi}=\sqrt{\mathbf{D}\xi}=\frac{1}{\lambda}.$$

$$\int\limits_0^{+\infty}x^2\lambda e^{-\lambda x}dx\hspace{10cm}.$$

.

$$\xi,\hspace{10cm}.$$

$$\hspace{1.5cm},\hspace{10cm},$$

$$\xi,\hspace{10cm},$$

.

$$\xi.$$

,

.

$$\sigma=\sqrt{\mathbf{D}\xi}\hspace{10cm}\xi.\hspace{10cm}\sigma$$

$$\xi.$$

1. $\xi \rightarrow \eta$. $(\xi + \eta)$.
2. ξ
 ξ , $2 \rightarrow 10$.
 $2\xi + 5$.
3. ξ

$$f(x)=\begin{cases}\frac{c}{x}, & x\in[1;2],\\0, & x\notin[1;2].\end{cases}$$

4. $\xi, \mathbf{D}\xi$
 ξ

$$f(x)=\begin{cases}\frac{\pi}{2}\cos^2x, & |x|<\frac{\pi}{2},\\0, & |x|>\frac{\pi}{2}.\end{cases}$$

$$\mathbf{M}\xi, \mathbf{D}\xi.$$

1.11.

$$\xi>0$$

$$P\Big(|\xi-\mathbf{M}\xi|\geq \xi\Big)\leq \frac{\mathbf{D}\xi}{\varepsilon^2}.$$

$$P\big(|\xi-\mathbf{M}\xi|<\xi\big)\geq 1-\frac{\mathbf{D}\xi}{\varepsilon^2}.$$

.

$$\cdot \, \xi_1, \, \xi_2, \ldots \xi_n \qquad \qquad \qquad ,$$

$$1). \qquad \qquad \qquad ;$$

$$2). \qquad \qquad \qquad ,$$

$$>0 \quad , \qquad \qquad \qquad \varepsilon>0,$$

$$\lim_{n\rightarrow\infty}P\bigg(\bigg|\frac{1}{n}\sum_{i=1}^n\xi_i-\frac{1}{n}\sum_{i=1}^nM\xi_i\bigg|<\varepsilon\bigg)=1.$$

$$\cdot \qquad \qquad \qquad \mu_n \quad - \qquad \qquad \qquad n$$

.

$$\varepsilon>0$$

$$\lim_{n\rightarrow\infty}P\bigg(\bigg|\frac{\mu_n}{n}-p\bigg|<\varepsilon\bigg)=1.$$

$$[5, \quad .\, 115\text{-}119; 9, \quad .\, 146\text{-}154; 13, \quad .\, 130\text{-}133].$$

$$1. \qquad \qquad \qquad , \qquad \qquad \qquad ,$$

$$\xi$$

.

$$\mathbf{D} \xi = \sigma^2, \varepsilon = 3\sigma,$$

$$P(|\xi - \mathbf{M}\xi| < 3\sigma) \geq 1 - \frac{\sigma^2}{9\sigma^2}, \dots$$

$$P(|\xi - \mathbf{M}\xi| < 3\sigma) \geq \frac{8}{9} \approx 0,89:$$

« ».

« » (1.7).

2. ξ

ξ	0,3	0,6
	0,2	0,8

,

$$|\xi - \mathbf{M}\xi| < 0,2$$

$$\xi, \quad \xi^2 \quad \mathbf{D}\xi = \xi^2 - (\xi)^2.$$

$$\xi = 0,3 \cdot 0,2 + 0,6 \cdot 0,8 = 0,06 + 0,48 = 0,54,$$

$$\xi^2 = 0,09 \cdot 0,2 + 0,36 \cdot 0,8 = 0,018 + 0,288 = 0,306,$$

$$\mathbf{D}\xi = 0,306 - 0,2916 = 0,0144.$$

$$P(|\xi - 0,54| < 0,2) \geq 1 - \frac{0,0144}{0,04} = 0,64.$$

ξ $\mathbf{D}\xi.$

\cdot ,
 $($
 n
 $)$

$$\frac{1}{n} \sum_{i=1}^n \mathbf{M}_{\xi_i}.$$

1. ,
 $($) 90 .
 0,0225.
 , ,)

0,4;) ,
 89,7 90,3.

2. $\xi_1, \xi_2, \dots \xi_n.$
 ξ ($n = 1, 2, \dots$)
 $: \sqrt{n}, 0, \sqrt{n}$, $\frac{1}{n}, 1 - \frac{2}{n}, \frac{1}{n}.$
 ?

2.

2.1.

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ξ —

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; $F(x)$ — $1, 2, \dots, n$ — ξ , n

ξ , n

i i - n m $y_i \left(i = \overline{1, m} \right)$

n_i , $\omega_i = n_i / n$

y_i . $\sum_{i=1}^m n_i = n$. $\sum_{i=1}^m \omega_i = 1$.

m n , k

$k \approx 1 + 3,322 \ln n$

$($ $)$

$h = \frac{x_{\max} - x_{\min}}{k}$

x_{\max} ; x_{\min} — $:$

(x_i, ω_i) .

$$\begin{aligned} & \left(x_i, x_{i+1}\right), \\ \omega_i & \quad n_i. \end{aligned}$$

2.2.

$$F(x)=P(\xi < x), \qquad \qquad \qquad , \qquad \qquad \qquad , \qquad \qquad \qquad .$$

$$F^*(x).$$

$$F^*(x)$$

$$F(x) \qquad \qquad \qquad ,$$

$$n_i/n, \quad . \quad .$$

$$F^*(x)=\sum_{x_i<x}\frac{n_i}{n}.\tag{2.1}$$

$$\begin{aligned} (2.1) \qquad \qquad \qquad & \qquad \qquad \qquad . \\ & [x_i, x_{i+1}] \qquad \qquad \qquad . \end{aligned}$$

$$\begin{aligned} & , \\ & , \\ & . \end{aligned}$$

$$1, \quad 2, \quad \cdots \quad n, \qquad \qquad n:$$

$$\overline{x}=\frac{1}{n}\sum_{i=1}^nx_i\,.$$

$$,$$

$$\overline{x}=\frac{1}{n}\sum_{i=1}^my_in_i,\tag{2.2}$$

$y_1, y_2, \dots, y_m, \dots, m$ ξ
 $n; n_i \left(i = \overline{1, m} \right) -$ $y.$
(2.2) y
 $,$ $-$ $.$
 $,$ \dots $,$

$$S^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 = \frac{1}{n-1} \left(\sum_{i=1}^n x_i^2 - n\bar{x}^2 \right).$$

$,$ $,$

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^m n_i y_i^2 - n\bar{x}^2 \right). \tag{2.3}$$

(2.3) (2.2).

[5, . 120-125; 10, . 181-187].

1. ():

1, 9, 6, 7, 7, 3, 5, 6, 6, 2, 4, 7, 8, 0, 9, 7, 5, 3, 5, 2, 6, 5, 4, 6, 9, 3, 10, 4.
1, 6, 7, 2, 4, 5, 4, 6, 9, 4, 2, 3, 5, 2, 10, 7, 2, 4, 5, 8, 8, 4. $n = 50.$

11

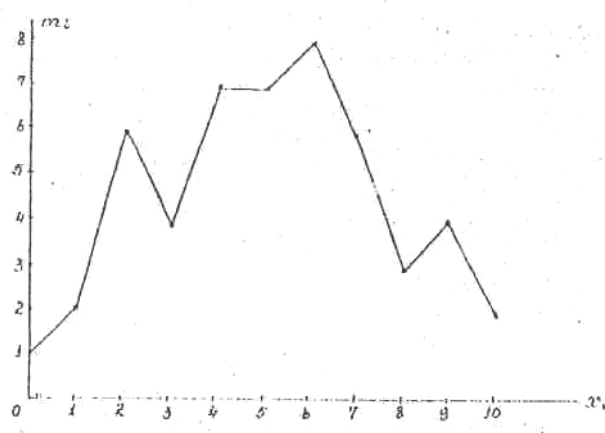
 $(m = 11):$

0, 1, 2, 2, 4, 5, 6, 7, 8, 9, 10.

,

y_i	0	1	2	3	4	5	6	7	8	9	10
n_i	1	2	6	4	7	7	8	6	3	4	2

:



2.

,

:

7,59; 7,48; 7,46; 7,40; 7,24; 7,41; 7,34; 7,43; 7,38; 7,60;
 7,26; 7,43; 7,37; 7,55; 7,42; 7,41; 7,30; 7,14; 7,42; 7,52;
 7,46; 7,39; 7,35; 7,32; 7,18; 7,30; 7,54; 7,38; 7,37; 7,34;
 7,50; 7,61; 7,42; 7,32; 7,36; 7,40; 7,67; 7,31; 7,40; 7,36;
 7,28; 7,58; 7,38; 7,58; 7,26; 7,37; 7,28; 7,39 7,32; 7,20;
 7,43; 7,34; 7,45; 7,33; 7,41; 7,33; 7,45; 7,31; 7,45; 7,39.

 $n = 60.$

.

$$1 + 3,32 \lg 60 \approx 6,90.$$

$r = 7$. $x_{\min} = 7,14$; $x_{\max} = 7,61$;
 $h = (7,61 - 7,14) : 7 = 0,067$.
 $0,01$,
 h .
 $($ $h = 0,08$. $x_1 = 7,10$,
 $($ $)$

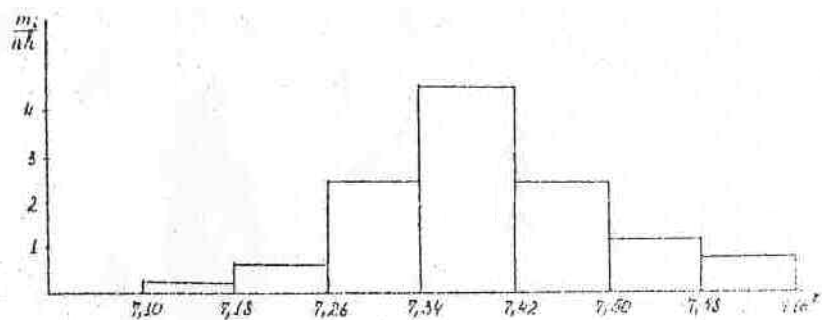
$$= x_1 + 7 \cdot h = 7,10 + 7 \cdot 0,08 = 7,66 > x_{\max} = 7,61.$$

x_{\min} x_{\max}

x_1, h, r .

$\frac{m_i}{nh}$

$(x_1 \quad x)$	(7,10;7,18)	(7,18;7,26)	(7,26;7,34)	(7,34;7,42)	(7,42;7,50)	(7,50;7,58)	(7,58;7,69)
x_i	7,14	7,22	7,30	7,36	7,46	7,54	7,62
	1	3	12	22	12	6	4
$\frac{m_i}{nh}$	0,21	0,63	2,5	4,59	2,50	1,25	0,83



3.

1.

(2.2),

$$\begin{aligned}\bar{x} &= \frac{1}{50} (0 \cdot 1 + 2 \cdot 1 + 6 \cdot 2 + 4 \cdot 3 + 7 \cdot 4 + 7 \cdot 5 + 8 \cdot 6 + 6 \cdot 7 + 3 \cdot 8 + 4 \cdot 9 + 2 \cdot 10) = \\ &= \frac{1}{50} \cdot 259 = 5,18\end{aligned}$$

(2.3)

$$\begin{aligned}S^2 &= \frac{1}{49} (1 \cdot 0 + 2 \cdot 1 + 6 \cdot 2^2 + 4 \cdot 3^2 + 7 \cdot 4^2 + 7 \cdot 5^2 + 8 \cdot 6^2 + \\ &+ 6 \cdot 7^2 + 3 \cdot 8^2 + 4 \cdot 9^2 + 2 \cdot 10^2) = \frac{1}{19} \cdot 305,38 \approx 6,23.\end{aligned}$$

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2.3.

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θ

$$\tilde{\theta}_n=\tilde{\theta}_n(X_1,X_2,...,X_n).$$

$$\tilde{\theta}_n(x_1,x_2,...,x_n)$$

$$X_1=x_1,\;X_2=x_2,\;\ldots\;X_n=x_n,$$

$$\theta.$$

,

$$\theta.$$

$$\tilde{\theta}_n$$

$$\theta$$

$$,\qquad\qquad\tilde{\theta}_n=\theta.$$

$$\tilde{\theta}_n$$

$$\theta$$

$$,\qquad\qquad\qquad n\rightarrow\infty$$

$$P\Big(\big|\tilde{\theta}_n-\theta\big|<\varepsilon\Big)\rightarrow 1\qquad\qquad\varepsilon>0.$$

,

.

$$[5,\quad.126-130;10,\quad.202-208].$$

1.

$$x_1,x_2,...,x_n$$

$\xi \qquad F_\xi(x)=P(\xi\leq x).$ $a=\mathbf{M}\xi.$

$\mathbf{D}\xi=\sigma^2$ (,

).

\overline{X} a

, . .

$$\tilde{a}_n=\overline{X}=\frac{x_1+x_2+...+x_n}{n}.$$

$X_1,X_2,...,X_n$

$\xi.$

$\mathbf{M}X_i=a,\mathbf{D}X_i=\sigma^2.$

\tilde{a}_n

,

$$\mathbf{M}\tilde{a}_n=\mathbf{M}X=\frac{1}{n}(\mathbf{M}X_1+\mathbf{M}X_2+...+\mathbf{M}X_n)=\frac{na}{n}=a,$$

$\tilde{a}_n=\overline{X}$

$\xi.$

$$\mathbf{D}\tilde{a}_n=\frac{1}{n^2}(\mathbf{D}X_1+\mathbf{D}X_2+...+\mathbf{D}X_n)=\frac{n\sigma^2}{n^2}=\frac{\sigma^2}{n}.$$

$P\bigl(\bigl|\tilde{a}_n-a\bigr|<\varepsilon\bigr)\geq 1-\frac{\mathbf{D}\tilde{a}_n}{\varepsilon^2}=1-\frac{\sigma^2}{n\varepsilon^2},$

$n\rightarrow\infty\quad P\bigl(\bigl|\tilde{a}_n-a\bigr|<\varepsilon\bigr)\rightarrow 1,\quad .\quad .\quad \quad \tilde{a}_n=\overline{X}\quad .$

$\sigma^2\qquad \sigma^2=S^2,$

$$S^2=\frac{1}{n-1}\sum_{i=1}^n\bigl(X_1-\overline{X}\bigr)^2,$$

$$S^2$$

$$\sigma^2$$

$$\xi^4,$$

$$n \rightarrow \infty \quad \mathbf{D}S^2 = 0\left(\frac{1}{n}\right),$$

$$S^2.$$

,

 .

 .

1.

ξ	1	2	3	4
n_i	20	15	10	5

$$)$$

$$($$

$$);$$

$$)$$

$$S^2.$$

2.

$$($$

$$)$$

$$($$

$$): \quad _1 = 92; \quad _2 = 94; \quad _3 = 103; \quad _4 = 105; \quad _5 = 106.$$

$$)$$

$$;$$

$$)$$

$$.$$

2.4.

$$($$

$$,$$

$$)$$

$$,$$

$$,$$

θ
 $\left[\theta_n^1;\theta_n^2\right]$
 θ

[5, . 133-136; 10, . 233-236; 13, . 176-180].

1.

$$\overline{X}=\frac{1}{n}\big(X_1+X_2+...+X_n\big)$$

$$X_i\;\; i=\overline{1,n}$$

$$(\;\;,\sigma),$$

$$\mathbf{M}\overline{X}=\mathbf{M}X_i=a\,,\quad \mathbf{D}\overline{X}=\frac{\sigma^2}{n}\,.$$

$$\frac{\overline{X}-a}{\sigma}\sqrt{n}$$

(0;1)

1

$$P\bigg(-u_{\gamma}\leq\frac{\overline{X}-a}{\sigma}\sqrt{n}\leq u_{\lambda}\bigg)=\gamma\,.\tag{2.4}$$

$$u_{\gamma} = 2,58. \quad (2.4)$$

$$P\left(\overline{X}-u_{\gamma} \frac{\sigma}{\sqrt{n}} \leq a \leq \overline{X}+u_{\gamma} \frac{\sigma}{\sqrt{n}}\right)=\gamma .$$

$$P\left(\overline{X}-t_{\gamma} \frac{S}{\sqrt{n-1}} \leq a \leq \overline{X}+t_{\gamma} \frac{S}{\sqrt{n-1}}\right)=\gamma . \quad (2.5)$$

$$S^2=\frac{1}{n-1} \sum_{i=1}^n\left(X_i-\overline{X}\right)^2 .$$

2. 10

1 :

	1	2	3	4	5	6	7	8	9	10
	25,4	28	20,1	27,4	25,6	23,9	24,8	26,4	27,0	25,4

$$\gamma=0,95.$$

$$\overline{X}=25,4; S=2,12. \quad (2.5).$$

$$k=1 \qquad t_{\gamma}=2,26,$$

$$t_{\gamma} \cdot \frac{S}{\sqrt{n-1}}=2,26 \cdot \frac{2,12}{\sqrt{9}}=1,6.$$

$$25,4-1,6 \leq \qquad \leq 25,4+1,6 \qquad 23,8 \leq \qquad \leq 27.$$

$$3. \qquad \qquad \qquad \xi \qquad \qquad \qquad , \quad \sigma_{\xi} = 1.$$

$$\overline{X}=2; n=16; \gamma=0,99.$$

$$1 \qquad \qquad u_{\gamma} = 2,58, \qquad u_{\gamma} \frac{\sigma}{\sqrt{n}}$$

$$2,58 \cdot \frac{1}{4} = 0,65.$$

$$2-0,65 \leq \qquad \leq 2+0,65 \qquad 1,35 \leq \qquad \leq 2,65.$$

$$[5, \quad . \, 133-136; 10, \quad . \, 233-236].$$

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 Δ.
 γ, Δ, n
 ,
 .
 n γ Δ. γ , Δ
 « » , ,
 .
 n, γ Δ.

$$1. \qquad - \qquad . \qquad \qquad \qquad n = 100$$

0,95 ,

$\sigma = 2$.

2. $n = 10$.

ξ	-2	1	2	3	4	5
n_i	2	1	2	2	2	1

$0,95 \text{ M}\xi = a$ σ

()

2.5.

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, . .

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0 ,

$\{H_\lambda\}$ $(\lambda$

).

,

H_λ

.

$A \subseteq X$, H_λ

$P(x \in A / H_\lambda)$, . . ,

H_λ .

H_0 ,

(H_0) S ($x \in S$),
 S
 $\alpha > 0$,
 $P(x \in S/H_0) \leq \alpha$.
 H_0 ,
 α , 0,05; 0,01
0,001.
 S ,
 $P(x \in S/H_0) \leq \alpha$ H_0 ($x \in S$).
 $P(x \in S/H_0) \leq \alpha$,
 α .
 H_0 ,
 H_λ .
 $\beta(\lambda) = P(\overline{x} \in S/H_\lambda) = 1 - P(x \in S/H_\lambda) = P(\bar{S}/H_\lambda)$.
 $1 - \beta(\lambda) = P(x \in S/H_\lambda)$,
 H_0 , H_λ ,
 S ,
 $P(x \in S/H_\lambda) \leq \alpha$ \tilde{S} ,
 $P(\tilde{S}/H_\lambda) = \max P(S/H_\lambda)$,
 $\max S P(x \in S/H_0) \leq \alpha$,

$$\begin{array}{ccccccc} \tilde{S}=S_\lambda & & \lambda. & & S_\lambda & & \lambda, \quad \tilde{S}=S_\lambda \\ & & & & & & \\ & & & & & & \tilde{S} \\ & & & & & & \\ & & & & & & \end{array}$$

$$\chi^2(\quad).$$

$$\begin{array}{ccc} H_0 & , & x_1,x_2,...x_n \quad n \\ & & F_0(x). \end{array}$$

$$\begin{array}{ccc} & H_0 & \chi^2, \\ & [7,10]. & k \\ h_1,h_2,...,h_k,\bigcup_{i=1}^kh_i=(-\infty;+\infty). & & P_i=P(x\in h_i),\;i=1,2,...,k. \end{array}$$

$$F_0(x). \qquad n_i \qquad h_i.$$

$$F_0(x)$$

$$\chi^2=\sum_{i=1}^k\frac{\left(n_i-nP_i\right)^2}{nP_i},\;n=\sum_{i=1}^kn_i\;.$$

$$\begin{array}{ccc} & & H_0 \\ & & \chi^2 \\ (k-1) & & \chi^2 \quad n\rightarrow\infty \end{array}$$

$$\chi^2_{\alpha, (k-1)} \quad (\alpha = 0,05 \quad \alpha = 0,01 \quad 0,001).$$

$$\chi^2_{\alpha,k-1}, \quad P(\chi^2 > \chi^2_{\alpha,k-1}) = \alpha.$$

$$\chi^2 = \sum_{i=1}^k \frac{(n_i - np_i)^2}{np_i}.$$

$$\chi^2 > \chi^2_{\alpha,k-1},$$

$$\alpha \quad H_0$$

$$\chi^2 \leq \chi^2_{\alpha,k-1},$$

$$H_0.$$

$$h_i (i = 1, 2, \dots, k)$$

$$np_i \geq 7.$$

$$F_0(x)$$

$$P_i = P(x \in h_i)$$

$$(k-1)$$

1.

2, 4, 2, 4, 3, 3, 3, 2, 0, 6, 1, 2, 3, 2, 2, 4, 3, 3, 5, 1
 0, 2, 4, 3, 2, 2, 3, 3, 1, 3, 3, 3, 1, 1, 2, 3, 1, 4, 3, 1
 7, 4, 3, 4, 2, 3, 2, 3, 3, 1, 4, 3, 4, 4, 5, 3, 4, 2, 4, 5
 3, 6, 4, 1, 3, 2, 4, 1, 3, 1, 0, 0, 4, 6, 4, 7, 4, 1, 3.

N = 79.

: 0.

7.

$\alpha = 0,01,$

$$\chi^2.$$

$$\lambda = \mathbf{M}\mathbf{X} = \mathbf{D}\mathbf{X},$$

$$\mathbf{M}\mathbf{X} \qquad \qquad \qquad :$$

$$\tilde{x} = \frac{\sum_{i=1}^m \frac{x_i - c}{k}}{n} k + c$$

$$\mathbf{D}\mathbf{X} \qquad \qquad \qquad :$$

$$S^2 = \frac{n}{n-1} \bar{S}^2,$$

$$\bar{S}^2 = \frac{\sum_{i=1}^m \left(\frac{x_i - c}{k}\right)^2 n_i}{n} k^2 - (\bar{x} - c)^2.$$

$$\bar{S}^2 -$$

$$S^2 -$$

$$\begin{array}{l} - \\ n - \\ m - \\ n - \\ k - \\ - \end{array} \qquad \qquad \qquad ,$$

$$\begin{array}{l} - \\ n - \\ m - \\ n - \\ k - \\ - \end{array} \qquad \qquad \qquad ,$$

$$\begin{array}{l} - \\ n - \\ m - \\ n - \\ k - \\ - \end{array} \qquad \qquad \qquad ,$$

$$\begin{array}{l} - \\ n - \\ m - \\ n - \\ k - \\ - \end{array} \qquad \qquad \qquad ($$

$$\begin{array}{l} - \\ n - \\ m - \\ n - \\ k - \\ - \end{array} \qquad \qquad \qquad ,$$

$$\begin{array}{l} - \\ n - \\ m - \\ n - \\ k - \\ - \end{array} \qquad \qquad \qquad \bar{x}, S^2$$

$$\begin{array}{l} - \\ n - \\ m - \\ n - \\ k - \\ - \end{array} \qquad \qquad \qquad = 3, k = 1.$$

$$\begin{array}{l} - \\ n - \\ m - \\ n - \\ k - \\ - \end{array} \qquad \qquad \qquad 1.$$

	n	$\frac{x_i - c}{k}$	$\frac{x_i - c}{k} n_i$	$\left(\frac{x_i - c}{k}\right)^2$	$\left(\frac{x_i - c}{k}\right)^2 n_i$
1	2	3	4	5	6
0	4	-3	-12	9	36
1	13	-2	-26	4	52
2	14	-1	-14	1	14
3	24	0	0	0	0
4	16	1	16	1	16
5	3	2	6	4	12
6	3	3	9	9	27
7	2	4	8	16	32
Σ	79	-	-13	-	189

$$\bar{x} = \frac{-13}{79} \cdot 1 + 3 = -0,16 + 3 = 2,84.$$

$$\bar{S}^2 = \frac{189}{79} \cdot 1 - (2,84 - 3)^2 = 2,3924 - 0,0256 = 2,3668.$$

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$$S^2 = \frac{79}{78} \cdot 2,3668 = 2,3971.$$

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 λ

2 3.

$$\lambda = 3.$$

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 $(\geq 5).$ $k = 6.$ χ^2

$$6 - 1 - 1 = 4 \quad (\lambda)).$$

$$P(\chi^2 > \chi_{0,01,4}^2) = 0,01.$$

2.

$$P_i \quad .$$

2

	n	P_i	nP_i	$n_i - nP_i$	$(n_i - nP_i)^2$	$\frac{(n_i - nP_i)^2}{nP_i}$
1	2	3	4	5	6	7
0	4	0,0498	3,9	0,1	0,01	0,0026
1	13	0,1494	11,8	1,2	1,44	0,1220
2	14	0,2240	17,7	-3,7	13,69	0,7734
3	24	0,2240	17,7	6,3	36,69	0,2424
4	16	0,1680	13,3	2,7	7,29	0,5481
≥ 5	8	0,1847	14,6	-6,6	43,56	2,9836
Σ	79	0,9999	79	-	-	6,6721

$$\chi^2$$

$$\chi_{0,01;4}^2 = 13,277$$

..

$$\chi^2 \geq 13,277.$$

$$6,6721, \quad \dots \quad \chi^2$$

,

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2.

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65	71	67	68	73	68	72	68	67	70	78	74	79	65	72
65	71	70	69	69	76	71	63	77	75	70	74	65	71	68
74	69	69	71	66	69	73	74	80	69	73	76	69	69	67
67	74	68	60	74	70	66	70	68	64	75	78	71	70	69
73	75	74	80	72	72	69	69	71	70	73	65	66	67	69
71	70	72	72	76	73	64	74	71	76	68	69	75	76	73
74	78	66	72	75	69	68	63	70	70	78	76	73	73	67
71	66	66	69	72	71	71	68	72	69	73	73	66	72	73
70	69	74	69	72	74	70	74	72	76	71	66	62	69	74
76	74	69	75	64	71	76	68	68	78	71	71	68	67	74
68	81	72	72	68	71	71	71	69	61	74	66	70	72	65
67	73	78	71	73	75	73	71	72	68	67	69	69	77	63
71	74	67	69	68	74	69	67	74	66	74	74	69	75	70
73	63	77	75	74										

$N = 200$.

59.

2.

$$\alpha = 0,1$$

,

.

$$\chi^2.$$

σ ,

$$a = \mathbf{M}(x), \quad \sigma = \sqrt{\mathbf{D}(x)}.$$

3.

		n_i	$\frac{x_i - c}{k}$	$\left(\frac{x_i - c}{k}\right)^2$	$\frac{x_i - c}{k} n_i$	$\left(\frac{x_i - c}{k}\right)^2 n_i$
1	2	3	4	5	6	7
59-61	60	1	-5	25	-5	25
61-63	62	2	-4	16	-8	32
63-65	64	7	-3	9	-21	63
65-67	66	16	-2	4	-32	64
67-69	68	27	-1	1	-27	27
69-71	70	40	0	0	0	0
71-73	72	38	1	1	38	38
73-75	74	38	2	4	76	152
75-77	76	18	3	9	54	162
77-79	78	9	4	16	36	144
79-81	80	3	5	25	15	75
81-83	82	1	6	36	6	36
Σ	—				132	818

$$= 70, k = 2.$$

$$\bar{x} = \frac{132}{200} \cdot 2 + 70 = 1,32 + 70 = 71,32.$$

$$\bar{S}^2 = \frac{818}{200} \cdot 4 - (71,32 - 70)^2 = 16,36 - 1,7424 = 14,6176.$$

$$\bar{x}, \bar{S}^2 \quad \mathbf{M}(x), \mathbf{D}(x), \quad :$$

$$\bar{x} = \frac{\sum_{i=1}^m \frac{x_i - c}{k}}{n} k + c,$$

$$\bar{S}^2 = \frac{\sum_{i=1}^m \left(\frac{x_i - c}{k}\right)^2 n_i}{n} k^2 - (\bar{x} - c)^2,$$

$$m - \sigma \quad \bar{S} = \sqrt{14,6176} = 3,82.$$

$$(\quad \cdot \quad \cdot 3)$$

, .

$$m = 8$$

$$\chi^2 \quad 8 - 2 - 1 = 5 \quad (\quad$$

$$(\sigma, \quad) \quad).$$

4.

4

	n _i	P _i	nP _i	n _i - nP _i	(n _i - nP _i)	$\frac{(n_i - nP_i)^2}{nP_i}$
1	2	3	4	5	6	7
− ∞ -65	10	0,04846	9,7	0,3	0,03	0,0093
65-67	16	0,08078	16,2	−0,2	0,04	0,0025
67-69	27	0,14501	29,0	−2,0	4,00	0,1379
69-71	40	0,19387	38,8	1,2	1,44	0,0371
71-73	38	0,20552	41,1	−3,1	9,61	0,2338
73-75	38	0,16034	32,1	5,9	34,81	1,0844
75-77	18	0,09921	19,8	−1,8	3,24	0,1636
77-∞	13	0,06681	13,4	−0,4	0,16	0,0119
Σ	200	1,0000	200,1			1,6805

i

$$P_i = \Phi\left(\frac{x_i - 71,32}{3.83}\right) - \Phi\left(\frac{x_{i-1} - 71,32}{3.83}\right),$$

$$x_{i-1} - \quad i- \quad, x_i - \quad i- \quad, \quad i = \overline{1,8},$$

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{t^2}{2}} dt -$$

$$P\big(\chi^2>\chi^2_{0,1;5}\big)=0,1.$$

$$\chi^2_{0,1;5}=9,236\quad\ldots$$

$$\chi^2\geq 9,236.$$

$$1,6805,\quad\ldots\quad\chi^2$$

,

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$$(\hspace{1.5cm})$$

,\hspace{0.5cm},

.

$$\{H_\lambda,\,\lambda\neq 0\}$$

$0,$

$$A\subset X,\hspace{1cm}X-$$

$$P\Big(x\in A\big/H_\alpha\Big).$$

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$$[7, \text{C. 63-65, C. 69; 1, C. 48; 10, C. 244-254}].$$

1.

«

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$$(\xi)\,12000\hspace{0.5cm}.$$

	1	2	3	4	5	6	7	8	9
	32	103	239	624	1187	1650	1883	1930	1638
	10	11	12	13	14	15	16		
	1130	737	417	221	110	57	32		

– ξ 7,00 ,
7,00–7,25
7,25–7,5 . .

0,01.

2.

ξ_i	0	1	2	3	4	5	6	7
n_i	112	168	130	68	32	5	1	1

ξ_i , –
 n_i , . . ,
 ξ_i ;

$$n = \sum n_i = 517.$$

$\alpha = 0,05$.

$$F=\frac{S_1^2}{S_2^2},$$

$$\begin{array}{l} S_1^2- \\ S_2^2- \end{array} \quad , \quad .$$

$$S_1^2 \quad S_2^2 \quad :$$

$$F=\frac{\frac{1}{m-1}\sum_{j=1}^m\big(\overline{x}_j-\overline{x}\big)^2n_j}{\frac{1}{n-m}\sum_{j=1}^m\sum_{i=1}^{n_j}\big(x_{ij}-\overline{x}_j\big)^2}$$

$$\begin{array}{l} F \qquad F - \qquad \qquad \qquad (\qquad - \qquad) \\ (m-1) \quad (n-m) (\qquad .4). \\ \qquad \qquad \qquad 0 (\qquad \qquad \qquad) \\ , \qquad \qquad \qquad : \end{array}$$

$$P(F>f_{\alpha})=\alpha\,,$$

$$\begin{array}{l} \alpha \qquad \qquad \qquad ; \qquad \qquad f_{\alpha} \quad - \qquad \qquad \qquad 4 \\ \qquad \qquad \qquad . \qquad \qquad \qquad F \qquad \qquad \qquad f_{\alpha}, \qquad \qquad \qquad 0 \\ \qquad \qquad \qquad , \quad . \quad . \\ \qquad \qquad \qquad . \\ 0, \quad . \quad . \qquad \qquad , \qquad \qquad \qquad . \end{array}$$

$$[7, \text{C. 83-86, 10, C. 294-306}].$$

1.

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).

1,2	1,1	1,5	1,6	1,2	1,5
1,0	1,4	1,0	1,4	1,2	2,0
0,8	1,1	1,1	1,5	1,8	1,9

$$\bar{x}_1 = 1; \quad \bar{x}_2 = \bar{x}_3 = 1,2; \quad \bar{x}_4 = 1,5; \quad \bar{x}_5 = 1,4; \quad \bar{x}_6 = 1,8.$$

$$\bar{x} = 1,35.$$

$$n_j = 3, j = 1, 6; n = 18 \quad m = 6.$$

$$0,2730, \quad - 0,0567.$$

$$4,81.$$

$$\alpha = 0,05. \quad 4, \quad , \quad f_\alpha = 3,11.$$

$$5 \quad 12$$

$$4,81, \quad ,$$

$$, \quad , \quad , \quad ,$$

$$, \quad .$$

$$5\%.$$

.

 F

.

$$Q = \sum_{j=1}^m \sum_{i=1}^{n_j} (x_{ij} - \bar{x})^2.$$

$$Q_1 = \sum_{j=1}^m \frac{1}{n_j} \left(\sum_{i=1}^{n_j} x_{ij} \right)^2 - \frac{1}{n} \left(\sum_{j=1}^m \sum_{i=1}^{n_j} x_{ij} \right)^2$$

$$Q_2=\sum_{j=1}^m\sum_{i=1}^{n_j}\bigl(x_{ij}-\overline{x_j}\bigr)^2,$$

$$Q=Q_1+Q_2.$$

$$S^2=\frac{Q}{n-1},\quad S_1^2=\frac{Q_1}{m-1},\quad S_2^2=\frac{Q_2}{n-m}$$

$$F.$$

$$Q=Q_1+Q_2,$$

$$Q_2=Q-Q_1.$$

$$y_{ij}=x_{ij}-C.$$

$$Q=Q_1.$$

$$Q=\sum_{j=1}^mP_j-\frac{1}{n}\biggl(\sum_{j=1}^mT_j\biggr)^2,$$

$$Q_1=\sum_{j=1}^m\frac{1}{n_j}(T_j)^2-\frac{1}{n}\biggl(\sum_{j=1}^mT_j\biggr)^2,$$

$$P_j=\sum_{i=1}^{n_j}y_{ij}^2,\quad T_j=\sum_{i=1}^{n_j}y_{ij}.$$

2.

:	F_1	F_2	F_3	F_4	F_5	F_6	F_7
1	75	104	96	92	76	92	89
2	86	89	88	89	89	87	85
3		92	105		90	88	93
4		90	90		77	82	
5		81	91		75	90	
6						86	

$$\alpha = 0,01$$

$$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = \mu_6 = \mu_7.$$

$$m = 7 \quad n_1 = n_4 = 2, n_2 = n_3 = n_5 = 5, n_6 = 6, n_7 = 3, \\ n_7 = 28.$$

$$\bar{x}$$

1.

1

	F_1	F_2	F_3	F_4	F_5	F_6	F_7	Σ
1	75	104	96	92	76	92	89	
2	86	89	88	89	89	87	85	
3		92	105		90	88	93	
4		90	90		77	82		
5		81	91		75	90		
6						86		
Σ	161	456	470	181	407	525	267	2467

$$\bar{x} = \frac{2467}{28} = 88,1$$

$$= 88$$

2

$i \backslash j$	F_1		F_2		F_3		F_4		F_5		F_6		F_7		Σ
	y_{i1}	y_{i1}^2	y_{i2}	y_{i2}^2	y_{i3}	y_{i3}^2	y_{i4}	y_{i4}^2	y_{i5}	y_{i5}^2	y_{i6}	y_{i6}^2	y_{i7}	y_{i7}^2	
1	-13	169	16	256	8	64	4	16	-12	144	4	16	1	1	
2	-2	4	1	1	0	0	1	1	1	1	-1	1	-3	9	
3			4	16	17	289			2	4	0	0	5	25	
4			2	4	2	4			-11	121	-6	36			
5			-7	49	3	9			-13	169	2	4			
6											-2	4			
T_j	-15		16		30		5		-33		-3				3
T_j^2 / n_j	112,5		51,2		180		12,5		217,8		1,5				578,5
P_j		173		326		366		17		439		61		35	1417

$$Q = \sum_{j=1}^m P_j - \frac{1}{n} \left(\sum_{j=1}^m T_j \right)^2.$$

$$Q = 1417 - \frac{9}{28} \approx 1416,7;$$

$$Q_1 = \sum_{j=1}^m \frac{1}{n_j} T_j^2 - \frac{1}{n} \left(\sum_{j=1}^m T_j \right)^2.$$

$$Q_1 = 578,5 - \frac{9}{28} \approx 578,2;$$

$$Q_2 = 1416,7 - 578,2 = 838,5.$$

$$S_1^2 = \frac{578,2}{7-1} = 96,4; \quad S_2^2 = \frac{838,5}{28-7} = 39,9;$$

$$F = \frac{96,4}{39,9} = 2,42.$$

$$m-1=6, \quad n-m=21$$

$\alpha = 0,01$
3,81.

4 F

$F >$

0

, ...

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3.

$\alpha = 0,05$

$$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4.$$

	F_1	F_2	F_3	F_4
1	-19	-31	-35	-31
2	-28	-33	-32	-27
3	-39	-35	-26	-28
4	-36	-25	-35	-35
5	-44	-28	-30	-40
6	-39	-31	-17	-41

$$m=4, n_j=6, j=\overline{1,4}, n=24.$$

3

	F_1	F_2	F_3	F_4
1	-19	-31	-35	-31
2	-28	-33	-32	-27
3	-39	-35	-26	-28
4	-36	-25	-35	-35
5	-44	-28	-30	-40
6	-39	-31	-17	-41
\bar{x}_j	-34,2	-30,5	-29,2	-32,0

(.3)

$$\bar{x} = \frac{-4,2 - 0,5 + 0,8 - 2,0}{4} - 30 = -31,5.$$

$$= -31.$$

4

$i \backslash j$	F_1		F_2		F_3		F_4		Σ
	y_{i1}	y_{i1}^2	y_{i2}	y_{i2}^2	y_{i3}	y_{i3}^2	y_{i4}	y_{i4}^2	
1	12	144	0	0	-4	16	0	0	
2	3	9	-2	4	-1	1	4	16	
3	-8	64	-4	16	5	25	3	9	
4	-5	25	6	36	-4	16	-4	16	
5	-13	169	3	9	1	1	-9	81	
6	-8	64	0	0	14	196	0	0	
T_j	-19		3		11		-6		-11
T_j^2 / n_j	60,2		1,5		20,2		6		87,9
P_j		475		65		255		122	917

$$Q = 917 - \frac{121}{24} \approx 917 - 5 = 912;$$

$$Q_1 = 87,9 - \frac{121}{24} \approx 87,9 - 5 = 82,9;$$

$$Q_2 = 912 - 82,9 = 829,1$$

$$S_1^2 = \frac{82,9}{4-1} = 27,63;$$

$$S_2^2 = \frac{829,1}{24} = 41,46;$$

$$F=\frac{27,63}{41,46}=0,67$$

$$m-1=3, \; n-m=20$$

$$F>3,10.$$

$$F$$

$$\alpha=0,05$$

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0
 .

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 F .
 ,

m ,
 .

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 ,
 , .

$$1). \qquad \qquad \qquad \alpha=0,05$$

$$_0\colon \mu_1=\mu_2=\mu_3=\mu_4.$$

.

	F_1	F_2	F_3	F_4
1	48	45	41	49
2	33	41	41	46
3	53	49	34	41
4	43	42	38	41
5	38	43	50	47
6	47	41	45	47

2).

 $\alpha = 0,01$ $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = \mu_6 = \mu_7.$

	F_1	F_2	F_3	F_4	F_5	F_6
1	89	92	91	107	103	113
2	106	105	101	99	104	109
3	88	94		76	74	89
4	102	92		101	97	99
5	107	100		101	103	92
6	89			96		100
7	101					

2.7.

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(1.9).

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$Y = y$ X
 , Y $y.$

,
 $\mathbf{M}(Y/X = x) = \mathbf{M}(X/Y = y).$

: $\mathbf{M}(Y/X = x)$

$Y,$

Y

.

,

$\mathbf{M}(X/Y = y)$

,

$y,$

$Y.$

..

$f(x)=\mathbf{M}(Y/X = x)$

Y

,

$g(y)=\mathbf{M}(X/Y = y)$

,

$Y.$

.

:

,

$\overline{x}_{Y=y} = \frac{1}{n_y} \sum_{i=1}^n yx_i n_{x_i y} ,$

$\overline{y}_{X=x} = \frac{1}{n_x} \sum_{i=1}^n xy_i n_{x_i y} ,$

, $Y.$

,

,

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,

, Y

:
.

r_{XY}

Y

:

$$r_{XY} = \frac{\mathbf{M}((X - \mathbf{M}X)(Y - \mathbf{M}Y))}{\sigma_x \cdot \sigma_y}.$$

:

1).

Y

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r_{XY}

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2).

$[-1;1]$.

3).

Y

, . . .

$$Y = aX + b,$$

$$r_{XY} = \begin{cases} +1, & a > 0. \\ -1, & a < 0. \end{cases}$$

$$, \dots r_{XY}^2 = 1,$$

$$Y = aX + b,$$

4).

$$r_{XY} \approx 1,$$

,

$$Y = aX + b,$$

,

.

$$|r_{XY}| < 0,5,$$

Y .

.

, . . .

$$y = \alpha_1 x + \alpha_0, \quad x = \beta_1 y + \beta_0.$$

$$\alpha_0, \alpha_1, \quad \beta_0, \beta_1$$

$$:$$

$$\alpha_0 = \bar{y} - \alpha_1 \bar{x}, \quad \alpha_1 = r \cdot \frac{\bar{S}_Y}{\bar{S}_X}.$$

$$\beta_0 = \bar{x} - \beta_1 \bar{y}, \quad \beta_1 = r \frac{\bar{S}_X}{\bar{S}_Y},$$

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i, \quad \bar{y} = \frac{1}{n} \sum_{i=1}^n y_i,$$

$$\overline{xy} = \frac{1}{n} \sum_{i=1}^n x_i y_i, \quad r = \frac{\overline{xy} - \bar{x} \cdot \bar{y}}{\bar{S}_x \cdot \bar{S}_y},$$

$$\bar{S}_x = \sqrt{\bar{S}_x^2}, \quad \bar{S}_y = \sqrt{\bar{S}_y^2},$$

$$\bar{S}_x^2 = \frac{1}{n} \sum_{i=1}^n x_i^2 - (\bar{x})^2, \quad \bar{S}_y^2 = \frac{1}{n} \sum_{i=1}^n y_i^2 - (\bar{y})^2$$

: [7, C. 76-83, 10, C. 313-333].

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$$u_i = \frac{x_i - c_1}{h_1} \quad v_j = \frac{y_j - c_2}{h_2}$$

$1, 2, \dots, h_1, h_2, \dots,$

$n_{xy} = n_{uv}, n_x = n_u, n_y = n_v.$

:

$$r = \left(\sum n_{uv}uv - \overline{nu}\overline{nv} \right) / n\sigma_u\sigma_v.$$
$$\sum n_{uv}uv$$

$$\sum n_{uv}uv = \sum vU,$$

$$U = \sum n_{uv}u,$$

$$\sum n_{uv}uv = \sum vV,$$

$$V = \sum n_{uv}v.$$

;

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1

Y	X						
	10	20	30	40	50	60	n_y
15	5	7	—	—	—	—	12
25	—	20	23	—	—	—	43
35	—	—	30	47	2	—	49
45	—	—	10	11	20	6	47
55	—	—	—	9	7	3	19
n_x	5	27	63	67	29	9	$n = 200$

$$u_i = \frac{(x_i - c_1)}{h_1}, \quad v_j = \frac{(y_j - c_2)}{h_2}.$$

$$= 40, \quad = 35,$$

$$: h_1 = 20 - 10 = 10, \quad h_2 = 25 - 15 = 10.$$

$$u_i = \frac{(x_i - 40)}{10}, \quad v_j = \frac{(y_j - 35)}{10}.$$

$$3$$

2

V	U						
	-3	-2	-1	0	1	2	n_v
-2	5	7	–	–	–	–	12
-1	–	20	23	–	–	–	43
0	–	–	30	47	2	–	79
1	–	–	10	11	20	6	47
2	–	–	–	9	7	3	19
n_u	5	27	63	67	29	9	$n = 200$

$$\sum n_{uv}uv$$

3.

1). $n_{uv} \neq 0$

$$5(-3) = -15;$$

$$7 \cdot (-2) = -14.$$

2).

3).

$$V \cdot U = (-2) \cdot (-29) = 58.$$

$$V = -2, \quad U = -29,$$

4).

$V \cdot U,$

$\sum_v vU,$

$\sum n_{uv}uv.$

3

$\sum_v vU = 169;$

$\sum n_{uv}uv = 169.$

3

V	U						$U =$ $= \sum n_{uv}u$	VU
	-3	-2	-1	0	1	2		
-2	<div><div></div><div>5</div><div></div></div>	<div><div></div><div>7</div><div></div></div>	-	-	-	-	-29	58
-1	-	<div><div></div><div>20</div><div></div></div>	<div><div></div><div>23</div><div></div></div>	-	-	-	-63	63
0	-	-	<div><div></div><div>30</div><div></div></div>	<div><div></div><div>47</div><div></div></div>	<div><div></div><div></div><div></div></div>	-	-28	0
1	-	-	<div><div></div><div>10</div><div></div></div>	<div><div></div><div>11</div><div></div></div>	<div><div></div><div>20</div><div></div></div>	<div><div></div><div>6</div><div></div></div>	22	22
2	-	-	-	<div><div></div><div>9</div><div></div></div>	<div><div></div><div>7</div><div></div></div>	<div><div></div><div>3</div><div></div></div>	13	26
$V =$ $\sum n_{uv}v$	-10	-34	-13	29	34	12		$\sum_v vU =$ $=169$
UV	30	68	13	0	34	24	$\sum_v uV =$ $=169$	<div><div></div><div></div><div></div></div>

:

$n_{uv}v,$

$n_{uv} \neq 0;$

$V;$

$u \quad V.$

$$\sum n_{uv}uv$$

$$\begin{aligned} & \cdot \\ & 3 \quad \bar{u} \quad \bar{v}, \\ & , \quad \bar{\sigma}_u \quad \bar{\sigma}_v \end{aligned}$$

$$\bar{\sigma}_u = \sqrt{\overline{u^2} - (\bar{u})^2}, \quad \bar{\sigma}_v = \sqrt{\overline{v^2} - (\bar{v})^2}.$$

$$\bar{u} = \frac{(\sum n_u u)}{n} = \frac{(5 \cdot (-3) + 27 \cdot (-2) + 63(-1) + 29 \cdot 1 + 9 \cdot 1)}{200} = -0,425$$

$$\bar{v} = \frac{(\sum n_v v)}{n} = \frac{(12 \cdot (-2) + 43(-1) + 47 \cdot 1 + 19 \cdot 2)}{200} = 0,09$$

$$\overline{u^2} = \frac{(\sum n_u u^2)}{n} = \frac{(5 \cdot 9 + 27 \cdot 4 + 63 \cdot 1 + 29 \cdot 1 + 9 \cdot 4)}{200} = 1,405$$

$$\bar{\sigma}_u = \sqrt{\overline{u^2} - (\bar{u})^2} = \sqrt{1,405 - (0,425)^2} = 1,106.$$

$$\bar{\sigma}_v = 1,209.$$

$$\sum n_{uv}uv = 169.$$

$$r = (\sum n_{uv}uv - n\bar{u}\bar{v})(n\bar{\sigma}_u\bar{\sigma}_v) = (169 - 200(-0,425) \cdot 0,09)/(200 \cdot 1,106 \cdot 1,209) = 0,603$$

$$r = 0,603.$$

2

Y *X*

$$2 \quad .$$

$$\bar{y}_x - \bar{y} = r \frac{\bar{\sigma}_y}{\bar{\sigma}_x} (x - \bar{x}).$$

$$\bar{\sigma}_x, \bar{\sigma}_y, \bar{x}, \bar{y}$$

$$\begin{aligned} \bar{\sigma}_x &= h_1 \bar{\sigma}_u, \quad \bar{\sigma}_y = h_{21} \bar{\sigma}_v, \quad \bar{x} = \bar{u} h_1 + c_1, \quad \bar{y} = \bar{v} h_2 + c_2, \\ . \quad . \quad \bar{u}, \bar{v}, \bar{\sigma}_u, \bar{\sigma}_v & \qquad \qquad \qquad r, \end{aligned}$$

$$\bar{x} = \bar{u} \cdot h_1 + c_1 = -0,425 \cdot 10 + 40 = 35,75.$$

$$\bar{y} = \bar{v} h_2 + c_2 = 0,09 \cdot 10 + 35 = 35,9.$$

$$\bar{\sigma}_x = \bar{\sigma}_u h_1 = 1,106 \cdot 10 = 11,06;$$

$$\bar{\sigma}_y = \bar{\sigma}_v h_2 = 1,209 \cdot 10 = 12,09.$$

$$Y \quad X$$

$$\bar{y}_x - 35,9 = 0,603 \frac{12,09}{11,06} (x - 35,75),$$

$$\bar{y}_x = 0,603x + 12,34.$$

$$\begin{aligned} Y(\quad X), \quad , \quad X(Y) \\ x(y). \end{aligned}$$

$\overline{\sigma}_{y/X=x}$
 $Y.$
 $\overline{\sigma}_{y/X=x}$
 Y
 $=$

$\overline{\sigma}_{x=Y=y}$
 $Y = y.$

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 \cdot
 $\cdot \cdot$
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$\cdot \cdot$

: [7, C. 76-83, 1, 45; 10, C. 313-333].

1). :

	2	4	6	8	10
Y	3,5	6	7	6	7,5

\cdot

2). ρ_1 , ρ_2
; - 79

$Y = \frac{\rho_1}{\rho_2}$
 $X :$

Y \ X	0,5	0,6	0,7	0,8
0,5	0	2	0	8
0,6	0	4	2	9
0,7	2	12	3	1
0,8	21	14	0	0
0,9	1	0	0	0

(x, y) ,
14
 $(0,6; 0,8)$
14
 X
 Y .

2.8.

$X(t)$
 t
(
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,
 t (
 T)
(
 t)
.
,
 $X(t_v)$
 t_v ,
 t («
 $t \gg$).
 $X(t)$
 t ,
.
,
 $X(t)$
 $x_p(t)$,
 $X(t)$
,
.
 $\{x_p(t)\}$
,
,
.

$$X(t) - \int_0^t X(t) dt.$$

$$F_1(x;t)=P(X(t)<x)$$

$$f_1(x;t)=\frac{dF_1(x;t)}{dx}.$$

$$, \qquad ,$$

$$x_1(t),...,x_n(t),...$$

$$P_1(t),...,P_n(t),...$$

$$P_i(t)=P(X(t)=x_i(t)), \quad \sum_i P_i(t)=1.$$

$$\begin{aligned} & t_1,t_2,...,t_n \text{ -- } t. \\ & (X(t_1),X(t_2),...,X(t_n)) \\ & X(t), \qquad t_i. \\ & n \qquad n \text{ -- } \\ & X(t). \\ & n \text{ -- } \qquad n \text{ -- } \end{aligned}$$

$$\begin{aligned} & F_n(x_1,x_2,...,x_n,;t_1,t_2,...,t_n,)= \\ & =P(X(t_1)<x_1,X(t_2)<x_2,...,X(t_n)<x_n), \end{aligned}$$

$$n \text{ -- }$$

$$f_n(x_1, x_2, \dots, x_n; t_1, t_2, \dots, t_n) = \\ = \frac{dF_n(x_1, x_2, \dots, x_n; t_1, t_2, \dots, t_n)}{dx_1 dx_2 \dots dx_n}.$$

$$X(t) = \sum_{n=1}^{\infty} \frac{f_n(x_1, x_2, \dots, x_n; t_1, t_2, \dots, t_n)}{n!}.$$

$$X(t) = \sum_{n=1}^{\infty} \frac{f_n(x_1, x_2, \dots, x_n; t_1, t_2, \dots, t_n)}{n!}.$$

$$X(t) = \sum_{n=1}^{\infty} \frac{f_n(x_1, x_2, \dots, x_n; t_1, t_2, \dots, t_n)}{n!}.$$

$$X(t) = \sum_{n=1}^{\infty} \frac{f_n(x_1, x_2, \dots, x_n; t_1, t_2, \dots, t_n)}{n!}.$$

$$X(t) = \sum_{n=1}^{\infty} \frac{f_n(x_1, x_2, \dots, x_n; t_1, t_2, \dots, t_n)}{n!}.$$

$$X(t) = \sum_{n=1}^{\infty} \frac{f_n(x_1, x_2, \dots, x_n; t_1, t_2, \dots, t_n)}{n!}.$$

$$X(t) = \sum_{n=1}^{\infty} \frac{f_n(x_1, x_2, \dots, x_n; t_1, t_2, \dots, t_n)}{n!}.$$

$$\begin{array}{c}
 \vdots \\
 1, \quad 2, \quad \ldots \quad n, \quad \ldots \\
 \\
 \\
 \\
 t_1, \, t_2, \, \ldots \, t_n, \, \ldots \\
 \\
 \\
 \\
 t_n \\
 \\
 j, \\
 \\
 \\
 \\
 t_{n-1} \qquad \qquad \qquad , \qquad \qquad \qquad (\\
) \qquad \qquad \qquad (\qquad \qquad \qquad .
 \end{array}$$

$$P_{ij}^{(n)}=P\left(E_j,t_n\Big/E_i,t_{n-1}\right)$$

$$\begin{array}{c}
 E_i \, (\qquad \qquad \qquad t_{n-1}) \qquad \qquad \qquad E_j \\
 \\
 t_n \qquad \qquad \qquad . \\
 \\
 \pi, \\
 \\
 P_{ij}^{(n)}
 \end{array}$$

$$\begin{array}{c}
 t_0=0. \\
 01, \qquad 02, \quad \ldots \quad 0k, \quad \ldots \\
 .
 \end{array}$$

$$\begin{array}{c}
 P_{ij}^{(n)} \qquad \qquad \qquad (\, . \, . \\
 \\
 P_{ij}^{(n)}=P_{ij}, \, \, n=1,3,\ldots), \qquad \qquad \qquad \pi
 \end{array}$$

$$\pi=\left(\begin{array}{ccccc} P_{11} & P_{12} & \ldots & P_{1k} & \ldots \\ P_{21} & P_{22} & \ldots & P_{2k} & \ldots \\ P_{k1} & P_{k2} & \ldots & P_{kk} & \ldots \\ - & - & - & - & - \end{array}\right), \quad \sum_j P_{ij}=1$$

$$\begin{array}{c}
 i. \qquad \qquad \qquad P_{ij}(m) \\
 \\
 \\
 \\
 \qquad \qquad \qquad m \\
 \\
 \ll \qquad \qquad \qquad \gg \qquad \qquad \qquad j. \qquad \qquad \qquad m
 \end{array}$$

$$\pi=\begin{pmatrix} P_{11}(m) & P_{12}(m) & \ldots & P_{1k}(m) & \ldots \\ P_{21}(m) & P_{22}(m) & \ldots & P_{2k}(m) & \ldots \\ - & - & - & - & - \end{pmatrix},$$

$$\pi_m=\pi^m.\qquad P_{ij}(m)$$

$$P_{ij}(m)=\sum_l P_{il}(S)P_{lj}(m-S),$$

$$S-[0;m].$$

$$P_j(n)=\sum_i P_{0i}P_{ij}(n)$$

$$t_n\qquad j\cdot$$

$$\begin{array}{l} (\\ ,\\ ,\\ \dots) \end{array}$$

$$\begin{array}{l} ,\\ ,\\ \qquad \qquad \qquad \ll \end{array}$$

$$\begin{array}{l} ,\\ :\\ 1). \qquad \qquad \qquad \cdot \qquad \qquad \qquad ; \end{array}$$

$$\begin{array}{l} k_1, \, k_2, \, \ldots \, k_n, \, \ldots \\ \qquad \qquad \qquad \cdot \qquad \qquad \qquad , \qquad \qquad \qquad k \quad - \\ (\quad , T+t) \qquad \qquad \qquad T \qquad \qquad \qquad k \quad t. \end{array}$$

$$\begin{array}{l} 2). \qquad \qquad \qquad \cdot \qquad \qquad \qquad , \\ \qquad \qquad \qquad k \end{array}$$

$$(\quad , T+t) \qquad \qquad \qquad ,$$

$$\qquad \qquad \qquad \cdot \qquad \qquad \qquad \cdot$$

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1.

1, 2, ..., 6,

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$$\frac{1}{4}$$

$$\frac{1}{2}.$$

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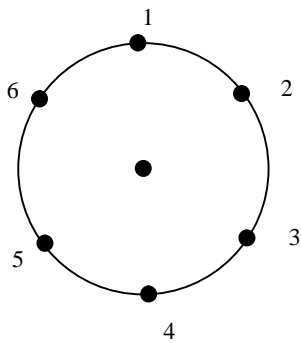
$$E_i, \ i=\overline{1,6}, \qquad \qquad \qquad . \ E_{ij}, \ i=\overline{1,6},$$

$$j=\overline{1,6}, \qquad \qquad \qquad i \neq j. \qquad \qquad \qquad :$$

$$E_{ii}=0, \ i=\overline{1,6}. \ E_{12}=\frac{1}{4}, \ E_{13}=0, \ E_{14}=\frac{1}{2}, \ E_{15}=0, \ E_{16}=\frac{1}{4}.$$

$$E_{21}=\frac{1}{4}, \ E_{23}=\frac{1}{4}, \ E_{24}=0, \ E_{25}=\frac{1}{2}, \ E_{26}=0 \quad . \ .$$

:



$$\begin{matrix} & \begin{pmatrix} E_1 & E_2 & E_3 & E_4 & E_5 & E_6 \end{pmatrix} \\ \begin{pmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \\ E_6 \end{pmatrix} & \begin{pmatrix} 0 & 1/4 & 0 & 1/2 & 0 & 1/4 \\ 1/4 & 0 & 1/4 & 0 & 1/2 & 0 \\ 0 & 1/4 & 0 & 1/4 & 0 & 1/2 \\ 1/2 & 0 & 1/4 & 0 & 1/4 & 0 \\ 0 & 1/2 & 0 & 1/4 & 0 & 1/4 \\ 1/4 & 0 & 1/2 & 0 & 1/4 & 0 \end{pmatrix} \end{matrix}$$

2.

$$P=\begin{pmatrix}0,4 & 0,6 \\ 0,3 & 0,7\end{pmatrix}$$

. . .

$$P_2=P^2, \quad . \quad . \quad P_2=\begin{pmatrix}0,4 & 0,6 \\ 0,3 & 0,7\end{pmatrix} \begin{pmatrix}0,4 & 0,6 \\ 0,3 & 0,7\end{pmatrix}=\begin{pmatrix}0,34 & 0,66 \\ 0,33 & 0,67\end{pmatrix}.$$

3.

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$$P_0 = (0001).$$

$$P_0(0) = P_1(0) = P_2(0) = 0, \quad P_3(0) = 1$$

$$i \geq j,$$

$$(i - j)$$

$$P_{ij} = c_i^{i-j} g^{i-j} (1 - g)^j.$$

$$P = \begin{pmatrix} E_0 & E_1 & E_3 & E_4 \\ E_0 & 1 & 0 & 0 \\ E_1 & g & 1 - g & 0 \\ E_2 & g^2 & 2g(1 - g) & (1 - g)^2 \\ E_3 & g^3 & 3g^2(1 - g) & 3g(1 - g)^2 \end{pmatrix} \begin{pmatrix} E_4 \\ 0 \\ 0 \\ 0 \\ (1 - g)^3 \end{pmatrix}.$$

$$P_l = P_{l-1}P, \quad l = 1, 2, \dots$$

$$P_1 = (0001) \begin{pmatrix} E_0 & E_1 & E_3 & E_4 \\ 1 & 0 & 0 & 0 \\ g & 1 - g & 0 & 0 \\ g^2 & 2g(1 - g) & (1 - g)^2 & 0 \\ g^3 & 3g^2(1 - g) & 3g(1 - g)^2 & (1 - g)^3 \end{pmatrix} =$$

$$= \begin{pmatrix} g^3 & 3g^2(1 - g) & 3g(1 - g)^2 & (1 - g)^3 \end{pmatrix}$$

4. .

- :
1). , ,
2). .
 ,
:

$$P = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{2}{5} & \frac{3}{5} \end{pmatrix} .$$

, .
 $P_0 = (1\ 0)$.
(,
).

$$P(1) = P(0) \cdot P = (1\ 0) \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{2}{5} & \frac{3}{5} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix} .$$

, n

$$P(n) = P(n - 1)P = P(0)P^n$$

$$P(n) = (P_1(n)P_2(n))$$

n	0	1	2	3	4	5
$P_1(n)$	1	0,5	0,45	0,445	0,4445	0,44445
$P_2(n)$	0	0,5	0,55	0,555	0,5555	0,55555

n ,

, . . . $P(0)=(01)$.

$P(n),\ n=\overline{0,5}$

n	0	1	2	3	4	5
$P_1(n)$	0	0,4	0,44	0,444	0,4444	0,44444
$P_2(n)$	1	0,5	0,56	0,556	0,5556	0,55556

, ,
 $n \rightarrow \infty; \ P_1(n) \rightarrow \frac{4}{9}; \ P_2(n) \rightarrow \frac{5}{9}, \ \dots$

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1.

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$\frac{1}{2}$,

$\frac{3}{8}$.

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: [10, C. 420-437; 13, . 163-173; C. 190-193].

$$\Phi(x) = \frac{2}{\sqrt{2\pi}} \int_0^x e^{-t^2/2} dt$$

	0	1	2	3	4	5	6	7	8	9
0,0	0,000	0080	0160	0238	0319	0399	0478	0558	0638	0717
0,1	0797	0876	0955	1034	1113	1192	1271	1350	1428	1507
0,2	1585	1663	1741	1819	1897	1974	2051	2128	2205	2282
0,3	2358	2434	2510	2586	2661	2737	2812	2886	2960	3035
0,4	3108	3182	3255	3328	3401	3473	3545	3616	3688	3759
0,5	3829	3899	3969	4039	4108	4177	4245	4313	4381	4448
0,6	4515	4581	4647	4713	4778	4843	4907	4971	5035	5098
0,7	5161	5223	5285	5346	5407	5467	5527	5587	5646	5705
0,8	5763	5821	5878	5935	5991	6047	6102	6157	6211	6265
0,9	6319	6372	6424	6476	6528	6579	6629	6679	6729	6778
1,0	6827	6875	6923	6970	7017	7063	7109	7154	7199	7243
1,1	7287	7330	7373	7415	7457	7499	7540	7580	7620	7660
1,2	7699	7737	7775	7813	7850	7887	7923	7959	7994	8029
1,3	8064	8098	8132	8165	8198	8230	8262	8293	8324	8355
1,4	8385	8415	8444	8473	8501	8529	8557	8584	8611	8638
1,5	8664	8690	8715	8740	8764	8789	8812	8836	8859	8882
1,6	8904	8926	8948	8969	8990	9011	9031	9051	9070	9090
1,7	9109	9127	9146	9164	9181	9189	9216	9233	9249	9265
1,8	9281	9297	9312	9327	9342	9357	9371	9385	9399	9412
1,9	9426	9439	9451	9464	9476	9488	9500	9512	9523	9534
2,0	9545	9556	9566	9576	9586	9596	9606	9616	9625	9634
2,1	9643	9651	9660	9668	9676	9684	9692	9700	9707	9715
2,2	9722	9729	9736	9743	9749	9756	9762	9768	9774	9780
2,3	9786	9791	9797	9802	9807	9812	9817	9822	9827	9832

[illegible]


 $t_{\gamma,k}$ —

k	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	0,95	0,98	0,99	0,999
1	0,16	0,32	0,51	0,73	1,00	1,38	1,96	3,08	6,31	12,71	31,82	63,66	636,6
2	14	29	44	62	0,82	06	34	1,89	2,92	4,30	6,96	9,92	31,6
3	14	28	42	58	76	0,98	25	64	35	3,18	4,54	5,84	12,94
4	13	27	41	57	74	94	19	53	13	2,78	3,75	4,60	8,61
5	13	27	41	56	73	92	16	48	01	57	36	03	6,86
6	0,13	0,26	0,40	0,55	1,72	1,91	1,13	1,44	1,94	2,45	3,14	3,71	5,96
7	13	26	40	55	71	90	12	41	89	36	00	50	5,41
8	13	26	40	55	70	89	11	40	86	31	2,90	35	5,04
9	13	26	40	54	70	88	10	38	83	26	82	25	4,78
10	13	26	40	54	70	88	09	37	81	23	76	17	4,59
11	0,13	0,26	0,40	0,54	0,70	0,88	1,09	1,36	1,80	2,20	2,72	3,11	4,49
12	13	26	39	54	69	87	08	36	78	18	68	05	4,32
13	13	26	39	54	69	87	08	35	77	16	65	01	4,22
14	13	26	39	54	69	87	08	34	76	14	62	2,98	4,14
15	13	26	39	54	69	87	07	34	75	13	60	95	4,07
16	0,13	0,26	0,39	0,53	0,69	0,86	1,07	1,34	1,75	2,12	2,58	2,92	4,02
17	13	26	39	53	69	86	07	33	74	11	57	90	3,97
18	13	26	39	53	69	86	07	33	73	10	55	88	3,92
19	13	26	39	53	69	86	07	33	73	09	54	86	3,88
20	13	26	39	53	69	86	06	32	72	09	53	84	3,85
21	0,13	0,26	0,39	0,53	0,69	0,86	1,06	1,32	1,72	2,08	2,52	2,83	3,82
22	13	26	39	53	69	86	06	32	72	07	51	82	3,79
23	13	26	39	53	68	86	06	32	71	07	50	81	3,77
24	13	26	39	53	68	86	06	32	71	06	49	80	3,75
25	13	26	39	53	68	86	06	32	71	06	48	79	3,73

26	0,13	0,26	0,39	0,53	0,68	0,86	1,06	1,31	1,71	2,06	2,48	2,78	3,71
27	13	26	39	53	68	85	06	31	70	05	47	77	3,69
28	13	26	39	53	68	85	06	31	70	05	47	76	3,67
29	13	26	39	53	68	85	05	31	70	04	46	76	3,66
30	13	26	39	53	68	85	05	31	70	04	46	75	3,65
40	0,13	0,25	0,39	0,53	0,68	0,85	1,05	1,30	1,68	2,02	2,42	2,70	3,55
60	13	25	39	53	68	85	05	30	67	00	39	66	3,46
120	0,13	0,25	0,39	0,53	0,68	0,84	1,04	1,29	1,66	1,98	2,36	2,62	3,37
∞	13	25	38	52	67	84	04	28	64	96	33	58	3,29

$$\chi^2 - \quad (\quad)$$

$$P(\chi^2 > x_\alpha) = \alpha$$

$\alpha \backslash k$	0,99	0,98	0,95	0,90	0,80	0,20	0,10	0,05	0,02	0,01
1	0,00016	0,0063	0,393	0,0158	0,6642	1,642	2,706	3,841	5,412	6,635
2	0,0201	0,0404	0,103	0,211	0,446	3,219	4,605	5,991	7,824	9,210
3	0,115	0,185	0,352	0,584	1,005	4,642	6,251	7,815	9,837	11,341
4	0,297	0,429	0,711	1,064	1,649	5,989	7,779	9,488	11,668	13,277
5	0,554	0,752	1,145	1,610	2,343	7,289	9,236	11,070	13,388	15,086
6	0,872	1,134	1,635	2,204	3,070	8,558	10,645	12,592	15,033	16,812
7	1,239	1,564	2,167	2,833	3,822	9,803	12,017	14,067	16,622	18,475
8	1,646	2,032	2,733	3,490	4,594	11,030	13,362	15,507	18,168	20,090
9	2,088	2,532	3,325	4,168	5,380	12,242	14,684	16,919	19,679	21,666
10	2,558	3,059	3,940	4,865	6,179	13,442	15,987	18,307	21,161	23,209
11	3,053	3,609	4,575	5,578	6,989	14,631	17,275	19,675	22,618	24,725
12	3,571	4,178	5,226	6,304	7,807	15,812	18,549	21,026	24,054	26,217
13	4,107	4,765	5,892	7,042	8,634	16,985	19,812	22,362	25,472	27,688
14	4,660	5,368	6,571	7,790	9,467	18,151	21,064	23,685	26,873	29,141
15	5,229	5,985	7,261	8,547	10,307	19,311	22,307	24,996	28,259	30,578
16	5,812	6,614	7,962	9,312	11,152	20,465	23,542	26,296	29,633	32,000
17	6,408	7,255	8,672	10,085	12,002	21,615	24,769	27,587	30,995	33,409
18	7,015	7,906	9,390	10,865	12,857	22,760	25,989	28,869	32,346	34,805
19	7,633	8,567	10,117	11,651	13,716	23,900	27,204	30,144	33,687	36,191
20	8,260	9,237	10,851	12,443	14,578	25,038	28,412	31,410	35,020	37,566
21	8,897	9,915	11,591	13,240	15,445	26,171	29,615	32,671	36,343	38,932
22	9,542	10,600	12,338	14,041	16,314	27,301	30,813	33,924	37,659	40,289
23	10,196	11,293	13,091	14,848	17,187	28,429	32,007	35,172	38,968	41,638
24	10,856	11,992	13,848	15,659	18,062	29,553	33,196	36,415	40,270	42,980
25	11,524	12,697	14,611	16,473	18,940	30,675	34,382	37,652	41,566	44,314
26	12,198	13,409	15,379	17,292	19,820	31,795	35,563	38,885	42,856	45,642
27	12,879	14,125	16,151	18,114	20,703	32,912	36,741	40,112	44,140	46,963
28	13,565	14,847	16,928	18,939	21,588	34,027	37,916	41,337	45,419	48,278
29	14,256	15,574	17,708	19,768	22,475	35,139	39,087	42,557	46,693	49,588
30	14,953	16,306	18,493	20,599	23,364	36,250	40,256	43,773	47,962	50,892

$\alpha = 0,01$

$k_1 \backslash k_2$	1	2	3	4	5	6	8	12	24	∞
1	4052	4999	5403	5625	5764	5859	5981	6106	6234	6366
2	98,49	99,00	99,17	99,25	99,30	99,33	99,36	99,42	99,46	99,50
3	34,12	30,81	29,46	28,71	28,24	27,91	27,49	27,05	26,60	26,12
4	21,20	18,00	16,69	15,98	15,52	15,21	14,80	14,37	13,93	13,46
5	16,26	13,27	12,06	11,39	10,97	10,67	10,29	9,89	9,47	9,02
6	13,74	10,92	9,78	9,15	8,75	8,47	8,10	7,72	7,31	6,83
7	12,25	9,55	8,45	7,85	7,46	7,19	6,84	6,47	6,07	5,65
8	11,26	8,65	7,59	7,01	6,63	6,37	6,03	5,67	5,28	4,86
9	10,56	8,02	6,99	6,42	6,06	5,80	5,47	5,11	4,73	4,31
10	10,04	7,56	6,55	5,99	5,64	5,39	5,06	4,71	4,33	3,91
11	9,65	7,20	6,22	5,67	5,32	5,07	4,74	4,40	4,02	3,60
12	9,33	6,93	5,95	5,41	5,06	4,82	4,50	4,16	3,78	3,36
13	9,07	6,70	5,74	5,20	4,86	4,62	4,30	3,96	3,59	3,16
14	8,86	6,51	5,56	5,03	4,69	4,46	4,14	3,80	3,43	3,00
15	8,68	6,36	5,42	4,89	4,56	4,32	4,00	3,67	3,29	2,87
16	8,53	6,23	5,29	4,77	4,44	4,20	3,89	3,55	3,18	2,75
17	8,40	6,11	5,18	4,67	4,34	4,10	3,79	3,45	3,08	2,65
18	8,28	6,01	5,09	4,58	4,25	4,01	3,71	3,37	3,00	2,57
19	8,18	5,93	5,01	4,50	4,17	3,94	3,63	3,30	2,92	2,49
20	8,10	5,85	4,94	4,43	4,10	3,87	3,56	3,23	2,86	2,42
21	8,02	5,78	4,87	4,37	4,04	3,81	3,51	3,17	2,80	2,36
22	7,94	5,72	4,82	4,31	3,99	3,76	3,45	3,12	2,75	2,31
23	7,88	5,66	4,76	4,26	3,94	3,71	3,41	3,07	2,70	2,26
24	7,82	5,61	4,72	4,22	3,90	3,67	3,36	3,03	2,66	2,21
25	7,77	5,57	4,68	4,18	3,86	3,63	3,32	2,99	2,62	2,17
26	7,72	5,53	4,64	4,14	3,82	3,59	3,29	2,96	2,58	2,13
27	7,68	5,49	4,60	4,11	3,78	3,56	3,26	2,93	2,55	2,10
28	7,64	5,45	4,57	4,07	3,75	3,53	3,23	2,90	2,52	2,06
29	7,60	5,42	4,54	4,04	3,73	3,50	3,20	2,87	2,49	2,03
30	7,56	5,39	4,51	4,02	3,70	3,47	3,17	2,84	2,47	2,01
40	7,31	5,18	4,31	3,83	3,51	3,29	2,99	2,66	2,29	1,80
60	7,08	4,98	4,13	3,65	3,34	3,12	2,82	2,50	2,12	1,60
120	6,85	4,79	3,95	3,48	3,17	2,96	2,66	2,34	1,95	1,38
∞	6,64	4,60	3,78	3,32	3,02	2,80	2,51	2,18	1,79	1,00

$$F - \quad (\quad)$$

$$P(F > f_{\alpha}) = \alpha \quad \alpha = 0,05$$

$k_1 \backslash k_2$	1	2	3	4	5	6	8	12	24	∞
1	161,4	199,5	215,7	224,6	230,2	234,0	238,9	243,9	249,0	254,3
2	18,51	19,00	19,16	19,25	19,30	19,33	19,37	19,41	19,45	19,50
3	10,13	9,55	9,28	9,12	9,01	8,94	8,84	8,74	8,64	8,53
4	7,71	6,94	6,59	6,39	6,26	6,16	6,04	5,91	5,77	5,63
5	6,61	5,79	5,41	5,19	5,05	4,95	4,82	4,68	4,53	4,36
6	5,99	5,14	4,76	4,53	4,39	4,28	4,15	4,00	3,84	3,67
7	5,59	4,74	4,35	4,12	3,97	3,87	3,73	3,57	3,41	3,23
8	5,32	4,46	4,07	3,84	3,69	3,58	3,44	3,28	3,12	2,93
9	5,12	4,26	3,86	3,63	3,48	3,37	3,23	3,07	2,90	2,71
10	4,96	4,10	3,71	3,48	3,33	3,22	3,07	2,91	2,74	2,54
11	4,84	3,98	3,59	3,36	3,20	3,09	2,95	2,79	2,61	2,40
12	4,75	3,88	3,49	3,26	3,11	3,00	2,85	2,69	2,50	2,30
13	4,67	3,80	3,41	3,18	3,02	2,92	2,77	2,60	2,42	2,21
14	4,60	3,74	3,34	3,11	2,96	2,85	2,70	2,53	2,35	2,13
15	4,54	3,68	3,29	3,06	2,90	2,79	2,64	2,48	2,29	2,07
16	4,49	3,63	3,24	3,01	2,85	2,74	2,59	2,42	2,24	2,01
17	4,45	3,59	3,20	2,96	2,81	2,70	2,55	2,38	2,19	1,96
18	4,41	3,55	3,16	2,93	2,77	2,66	2,51	2,34	2,15	1,92
19	4,38	3,52	3,13	2,90	2,74	2,63	2,48	2,31	2,11	1,88
20	4,35	3,49	3,10	2,87	2,71	2,60	2,45	2,28	2,08	1,84
21	4,32	3,47	3,07	2,84	2,68	2,57	2,42	2,25	2,05	1,81
22	4,30	3,44	3,05	2,82	2,66	2,55	2,40	2,23	2,03	1,78
23	4,28	3,42	3,03	2,80	2,64	2,53	2,38	2,20	2,00	1,76
24	4,26	3,40	3,01	2,78	2,62	2,51	2,36	2,18	1,98	1,73
25	4,24	3,38	2,99	2,76	2,60	2,49	2,34	2,16	1,96	1,71
26	4,22	3,37	2,98	2,74	2,59	2,47	2,32	2,15	1,95	1,69
27	4,21	3,35	2,96	2,73	2,57	2,46	2,30	2,13	1,93	1,67
28	4,20	3,34	2,95	2,71	2,56	2,44	2,29	2,12	1,91	1,65
29	4,18	3,33	2,93	2,70	2,54	2,43	2,28	2,10	1,90	1,64
30	4,17	3,32	2,92	2,69	2,53	2,42	2,27	2,09	1,89	1,62
40	4,08	3,23	2,84	2,61	2,45	2,34	2,18	2,00	1,79	1,52
60	4,00	3,15	2,76	2,52	2,37	2,25	2,10	1,92	1,70	1,39
120	3,92	3,07	2,68	2,45	2,29	2,17	2,02	1,83	1,61	1,25
∞	3,84	2,99	2,60	2,37	2,21	2,09	1,94	1,75	1,52	1,00

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$f(x)$	0	1	2	3	4	5	6	7	8	9
0,0	39894	39892	39886	39876	39862	39844	39822	39797	39767	39733
0,1	39695	39654	39608	39559	39505	39448	39387	39322	39253	39181
0,2	39104	39024	38940	38853	38762	38667	38568	38466	38361	38251
0,3	38139	38023	37903	37780	37654	37524	37391	37255	37115	36973
0,4	36827	36678	36526	36371	36213	36053	35889	35723	35553	35381
0,5	35207	35029	34849	34667	34482	34294	34105	33912	33718	33521
0,6	33322	33121	32918	32713	32506	32297	32086	31874	31659	31443
0,7	31225	31006	30785	30563	30339	30114	29887	29659	29430	29200
0,8	28969	28737	28504	28269	28034	27798	27562	27324	27086	26848
0,9	26609	26369	26129	25888	25647	25406	25164	24923	2468,1	24439
1,0	24197	23955	23713	23471	23230	22988	22747	22506	22265	22025
1,1	21785	21546	21307	21069	20831	20594	20327	20121	19886	19652
1,2	19419	19186	18954	18724	18494	18265	18037	17810	17585	17360
1,3	17137	16915	16694	16474	16256	16038	15822	15608	15395	15183
1,4	14973	14764	14556	14350	14146	13943	13742	13542	13344	13147
1,5	12952	12758	12566	12376	12188	12001	11816	11632	11450	11270
1,6	11092	10915	10741	10567	10396	10226	10059	09893	09728	09566
1,7	09405	09246	09089	08933	08780	08628	08478	08329	08183	08038
1,8	07895	07754	07614	07477	07341	07206	07074	06943	06814	06687
1,9	06562	06438	06316	06195	06077	05959	05844	05730	05618	05508
2,0	05399	05292	05186	05082	04980	04879	04780	04682	04586	04491
2,1	04398	04307	04217	04128	04041	03955	03871	03788	03706	03626
2,2	03547	03470	03394	03319	03246	03174	03103	03034	02965	02898
2,3	02833	02768	02705	02643	02582	02522	02463	02406	02349	02294
2,4	02239	02186	02134	02083	02033	01984	01936	01888	01842	01797
2,5	01753	01709	01667	01625	01585	01545	01506	01468	01431	01394
2,6	01358	01323	01289	01256	01223	01191	01160	01130	01100	01071
2,7	01042	01014	00987	00961	00935	00909	00885	00861	00837	00814
2,8	00792	00770	00748	00727	00707	00687	00668	00649	00631	00613
2,9	00595	00578	00562	00545	00530	00514	00499	00485	00470	00457
3,0	00443	00430	00417	00405	00393	00381	00370	00358	00348	00337
3,1	00327	00317	00307	00298	00288	00279	00271	00262	00254	00246
3,2	00238	00231	00224	00216	00210	00203	00196	00190	00184	00178
3,3	00172	00167	00161	00156	00151	00146	00141	00136	00132	00127
3,4	00123	00119	00115	00111	00107	00104	00100	00097	00094	00090
3,5	00087	00084	00081	00079	00076	00073	00071	00068	00066	00063
3,6	00061	00059	00057	00055	00053	00051	00049	00047	00046	00044
3,7	00042	00041	00039	00038	00037	00035	00034	00033	00031	00030
3,8	00029	00028	00027	00026	00025	00024	00023	00022	00021	00021
3,9	00020	00019	00018	00018	00017	00016	00016	00015	00014	00014
4,0	00013	00009	00006	00004	00002	00002	00001	00001	00000	00000

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{t^2}{2}} dt$$

$f(x)$	0	1	2	3	4	5	6	7	8	9
0,0	50000	50399	50798	51197	51595	51994	52392	52790	53188	53586
0,1	53983	54380	54776	55172	55567	55962	56356	56749	57142	57535
0,2	57926	58317	58706	59095	59483	59871	60257	60642	61026	61409
0,3	61791	62172	62552	62930	63307	63683	64058	64431	64803	65173
0,4	65542	65910	66276	66640	67003	67364	67724	68082	68439	68793
0,5	69146	69497	69847	70194	70540	70884	71226	71566	71904	72240
0,6	72575	72907	73237	73565	73891	74215	74537	74857	75175	75490
0,7	75804	76115	76424	76730	77035	77337	77637	77935	78230	78524
0,8	78814	79103	79389	79673	79955	80234	80511	80785	81057	81327
0,9	81594	81859	82121	82381	82639	82894	83147	83398	83646	83891
1,0	84134	84375	84614	84850	85083	85314	85543	85769	85993	86214
1,1	86433	86650	86864	87076	87286	87493	87698	87900	88100	88298
1,2	88493	88686	88877	89065	89251	89435	89617	89796	89973	90147
1,3	90320	90490	90658	90824	90988	91149	91308	91466	91621	91774
1,4	91924	92073	92220	92364	92507	92647	92786	92922	93056	93189
1,5	93319	93448	93574	93699	93822	93943	94062	94179	94295	94408
1,6	94520	94630	94738	94845	94950	95053	95154	95254	95352	95449
1,7	95543	95637	95728	95818	95907	95994	96080	96164	96246	96327
1,8	96407	96485	96562	96638	96712	96784	96856	96926	96995	97062
1,9	97128	97193	97257	97320	97381	97441	97500	97558	97615	97670
2,0	97725	97778	97831	97882	97932	97982	98030	98077	98124	98169
2,1	98214	98257	98300	98341	98382	98422	98461	98500	98537	98574
2,2	98610	98645	98679	98713	98745	98778	98809	98840	98870	98899
2,3	98928	98956	98983	99010	99036	99061	99086	99111	99134	99158
2,4	99180	99202	99224	99245	99266	99286	99305	99324	99343	99361
2,5	99379	99396	99413	99430	99446	99461	99477	99492	99506	99520
2,6	99534	99547	99560	99573	99585	99598	99609	99621	99632	99643
2,7	99653	99664	99674	99683	99693	99702	99711	99720	99728	99736
2,8	99744	99752	99760	99767	99774	99781	99788	99795	99801	99807
2,9	99813	99819	99825	99831	99836	99841	99846	99851	99856	99861
3,0	99865	99869	99874	99878	99882	99886	99889	99893	99896	99900
3,1	99903	99906	99910	99913	99916	99918	99921	99924	99926	99929
3,2	99931	99934	99936	99938	99940	99942	99944	99946	99948	99950
3,3	99952	99953	99955	99957	99958	99960	99961	99962	99964	99965
3,4	99966	99968	99969	99970	99971	99972	99973	99974	99975	99976
3,5	99977	99978	99978	99979	99980	99981	99981	99982	99983	99983
3,6	99984	99985	99985	99986	99986	99987	99987	99988	99988	99989
3,7	99989	99990	99990	99990	99991	99991	99992	99992	99992	99992
3,8	99993	99993	99993	99994	99994	99994	99994	99995	99995	99995
3,9	99995	99995	99996	99996	99996	99996	99996	99996	99997	99997
4,0	99997	99998	99999	99999	99999	—	—	—	—	—

[illegible]

[illegible]

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